How a Bohmian field theory could look

Thomas Benda

Institute of Philosophy of Mind
National Yang Ming University
Taipei, Taiwan

1. Einstein field equation

$$g = e$$

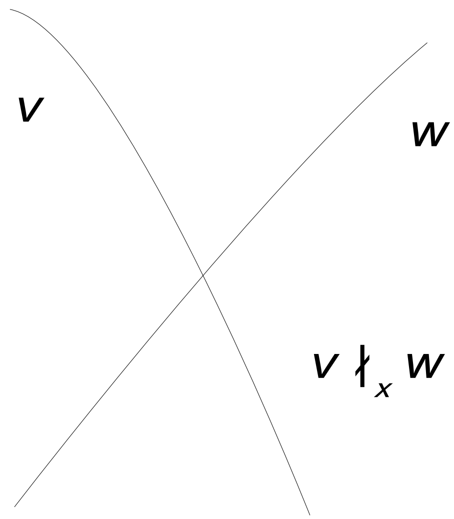
function of metric tensor (with or without cosmological constant)	energy-momentum tensor

We cannot take either side as given.

A tempting option: Define *e* by *g*, i.e., geometrize.
But: Physics is more than geometry by virtue of dynamics.
What dynamics is is a matter of dispute, it may very well involve experience.
g does not involve experience, if anything does, it has to be *e*.

2. Construction of *g*

In a (previously proposed) first-order theory ST, which is a conservative extension of Zermelo's Z with urelemente.



Worldlines	urelemente
Spacetime points	sets of worldlines
Spacetime manifold	
Connection	
Metric	

3. Constructing the energy-momentum tensor e

Some simple examples of e: single bodies, Schwartzschild, fluid, dust.

Dust: $e_{\mu\nu} = \rho \bullet u_\mu \bullet u_\nu$
(ρ: density; u: four-velocity)

We propose ρ and u to be derived from a plurality of wave functions.

For each worldline w, there is precisely one smooth wave function $\psi_w : \mathbb{R}^4 \to \mathbb{C}$. The domain of ψ_w is a configuration space s_w without metric.

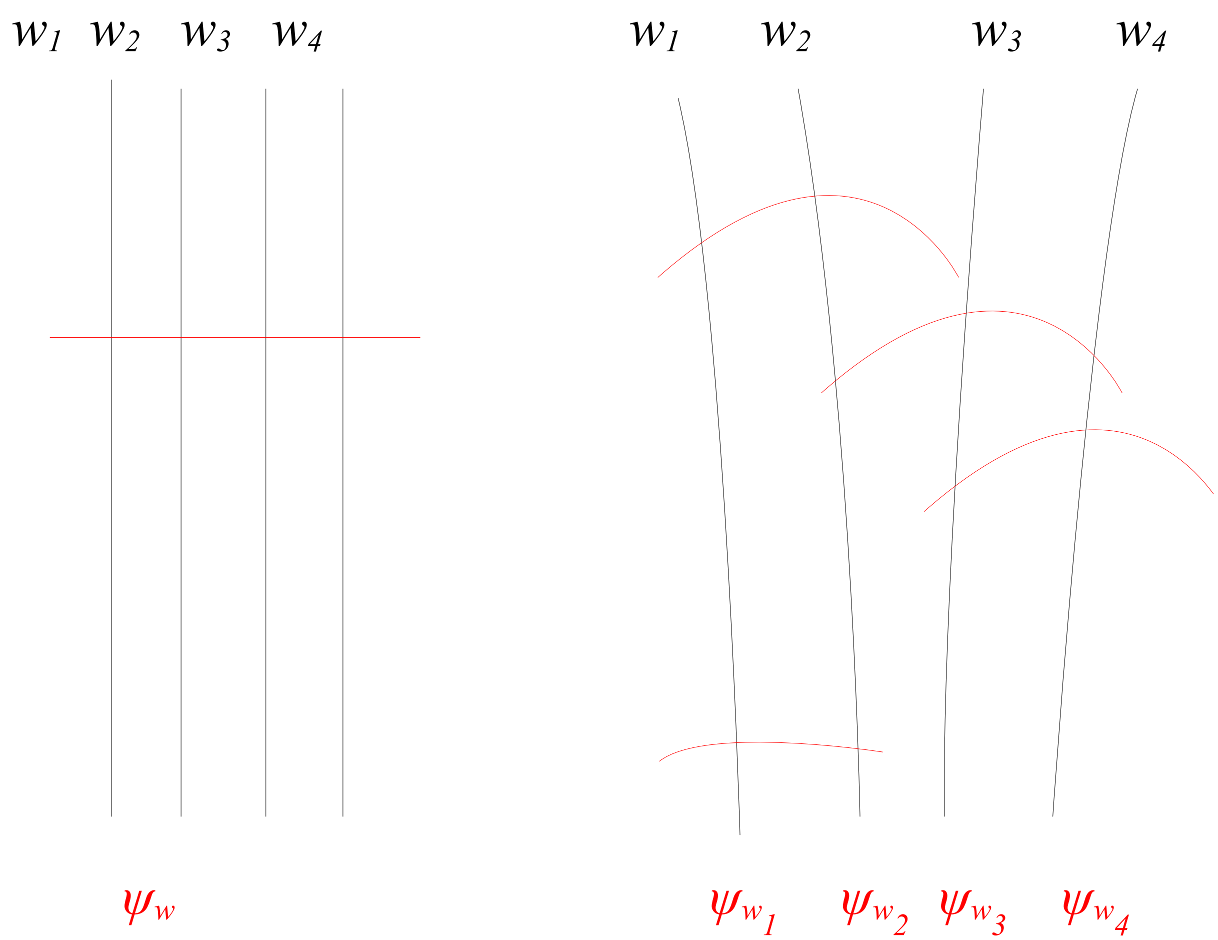
Setting up the dynamics:

The wave functions have a bearing on *e* in the following sense.

We define a density $\rho_w\!:\mathbb{R}\rightarrow\mathbb{R}$, $\rho_w(t) = \psi_w(t, 0)^{*}\bullet\psi_w(t, 0)$;
and a four-velocity $u_w\!:\mathbb{R}\rightarrow\mathbb{R}$, $u_w(t) = \mathrm{Im}\,(\psi_w(t, 0)^{*}\bullet\nabla\psi_w(t, 0) \,/\, \rho_w(t))$.

In the spacetime manifold with metric *g*: for every pair $\langle w, t\rangle$, there is a spacetime point $\mathrm{pnt}_w\, t$. Due to intersection of worldlines, the corresponding mapping is not one-to-one.
We stipulate $(\Lambda w_1 w_2 t_1 t_2\, (\mathrm{pnt}_{w_1}\, t_1 = \mathrm{pnt}_{w_2}\, t_2 \;\rightarrow\; \psi_{w_1}(t_1) = \psi_{w_2}(t_2))$, for which there is a model.

With that, writing stp for spacetime,we define $\rho\!:\mathrm{stp}\rightarrow\mathbb{R}$, such that
$\rho(\mathrm{pnt}_w\, t) = \rho_w(t)$ and
$u\!:\mathrm{stp}\rightarrow\mathbb{R}^4$, such that $u(\mathrm{pnt}_w\, t) = u_w(t)$.



Thus e is determined by all wave functions.

Simple solutions:

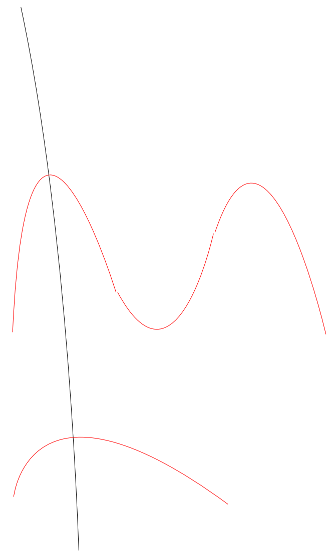
1. For all worldlines w, ψ_w are plane waves with the same frequency and wavelength, so each ρ_w is constant in s_w, ρ is constant throughout spacetime, so is g.

2. Roughly as before, but for some worldline w_1, ψ_{w_1} is a plane wave with higher frequency. A solution of the field equation $g = e$ will be an analogue of the one-body solution, i.e., a single body moving along a geodesics. In our model, ρ will be largely constant throughout spacetime, but higher along the worldline curve that corresponds to w_1.

Models of our theory include solutions of the Schrödinger equation as wave functions. We have not specified how to obtain, for every worldline w, its wave function ψ_w.

Why "Bohmian" (besides the dynamic equation to obtain *u*)?

After all, we haven't mentioned any particles. The wave functions are intended not to collapse, but to split into several components that are separated in their respective configuration spaces, as in Bohmian mechanics.



In contrast to Bohmian mechanics, we intend not a particle position to be experienced, but the splitting off of some part of the wave function.

4. Determining the wave functions

The wave functions ψ_w are determined within an axiomatic framework.

We propose an axiomatic first-order theory SB, which is a conservative extension of ST.

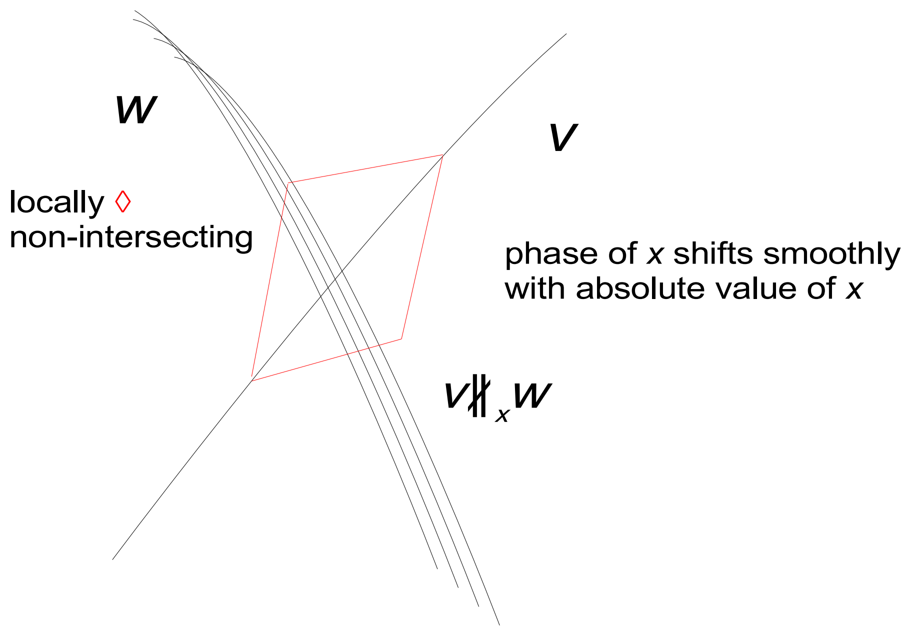
The language of contains the elementary formulas

$x = y \qquad x \in y \qquad x \nparallel_z y$ (“y intersects x at z”)

Axioms of SB, as added to or modified from ST:

(P1) $\bigwedge vwx\,(v \nparallel_x w \rightarrow \mathrm{W}v \wedge \mathrm{W}w \wedge x \in \mathbb{C})$
"*w* intersects *v* at *x* implies that *v*, *w* are worldlines and *x* is a complex number."

Writing avl *x* and pha *x* for the absolute value and phase of complex number *x*;
$v \nmid_x y$ for $\bigvee y\,(x = \mathrm{avl}\; y \wedge v \nparallel_y w)$;
$\mathrm{ifn}_z\, v$ for $\{x \mid \bigvee w\,(x = \langle \mathrm{avl}\; x, \mathrm{pha}\; x\rangle) \wedge w \in z \wedge v \nparallel_x w\}$
"the intersection location function of *v* through *z*",



(P11) $\bigwedge ux\,(\mathrm{W}u \wedge x \in \mathbb{C} \rightarrow \bigvee ab\,(a < x < b \wedge$

$\wedge \bigwedge u_1\,(\mathrm{W}u_1 \wedge (u \nmid_x u_1 \vee u = u_1) \rightarrow \bigvee! z\,(u_1 \in z \wedge$

$\wedge \bigwedge v{\in}z\,(\bigvee wy\,(\mathrm{Dmd}_{uab}\, wy \wedge (w \nmid_y v \vee w = v)) \wedge$

$\wedge \bigwedge wy\,(\mathrm{Dmd}_{uab}\, wy \rightarrow \bigvee! v{\in}z\,(w \nmid_y v \vee w = v)) \wedge$

$\wedge \bigwedge vwtsx_1x_2y_1y_2\,(v \in z \wedge w \in z \wedge \mathrm{W}t \wedge \mathrm{W}s \wedge \mathrm{Dmd}_{uab}\, vy_1 \wedge$

$\wedge \mathrm{Dmd}_{uab}\, wy_2 \wedge \mathrm{Isn}_{tv}\, x_1y_1 \wedge \mathrm{Isn}_{tw}\, x_2y_2 \rightarrow$

$\rightarrow \mathrm{dpb}_{st}\, x_1 / \mathrm{dpb}_{sv}\, y_1 = \mathrm{dpb}_{st}\, x_2 / \mathrm{dpb}_{sw}\, y_2) \wedge$

$\wedge \bigwedge v\,(\mathrm{Smo}\,(\mathrm{ifn}_z\, v))\,))))$ [only this line is new]

We now devise a way of, given v, coding the wave function ψ_v by $\{x \mid \bigvee w\,(v \nparallel_x w\}$, the set of locations at which some w intersects v.

Ideally, thereby some (non-gravitational) physics is reflected, but currently, we limit ourselves to simply providing some coding.

(P12) $\bigwedge vx \, \mathsf{Fin} \, \{w \mid v \nparallel_x w\}$

"For all v and x, the number of w intersecting v at x is finite."

A way to code the wave function ψ_v in the language of SB:
By (P12), given v and avl x and writing car z for the cardinality of z, the integral of car $\{w \mid v \nparallel_x w\}$ over pha x exists as do the integrals of momenta of car $\{w \mid v \nparallel_x w\}$ over pha x. Then, given v, and with $t \in \mathbb{R}$, $x \in \mathbb{R}^3$, we write $\psi_v(t, x)$ as a power series around the t-axis whose coefficients are above integrals. From there, we obtain, as outlined above, the energy momentum tensor e. Finally:

(P13) $g = e$

the Einstein field equation.

We envisage further axioms to determine the wave functions.

5. Conclusion

A simple field theory in relativistic spacetime has been sketched which is first order and axiomatic. It follows the spirit of Bohmian mechanics by having simple models that are approximated by one-particle Bohmian mechanics.

From here, it is hoped, an extension of relativistic spacetime to a physical theory can be developed.