LOGICS FOR REASONING ABOUT QUANTUM INFORMATION: A DYNAMIC-EPISTEMIC PERSPECTIVE

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Overview

Dynamic Epistemic Logic (DEL)

In standard Dynamic Epistemic Logic we work with **modal logics** to represent the information states of classical agents and we use **model-transforming operators** to represent the information changes.

Dynamic Quantum Logic

In **dynamic Quantum Logic** we work with dynamic logics (PDL-style) to represent quantum information and we use **state-transforming operators** to represent the quantum information changes.

Quantum DEL

Represent both what agents know after an action happens (this action includes e.g. the act of a quantum measurement) and the effect of this action on the state of the physical system.

Quantum Logic

What logical structure do we find in QM? (G. Birkhoff and J. von Neumann 1936)

- **Quantum Logic** = the propositional logic of the family of *closed linear subspaces* of the state space S (given by a Hilbert space $\mathcal{H}$).
- **OLD Approach:** A non-boolean structure of quantum propositions: a non-distributive or partial/fuzzy logic.

Dynamic Quantum Logic : No need to drop any classical logical principle!

- Add dynamic modalities to classical logic, expressing the *potential effect of actions*, e.g. *quantum measurements*.
- IDEA = **All the "non-classicality" of QL is due to actions**! To be captured by dynamic modalities.
- Expression $[\varphi?]\psi$ captures: after any sucessful test of φ, the system satisfies ψ.

Adding a "spatial"-part for compound quantum sytems

Compound systems

To capture the logical structure of compound quantum systems, we use "spatial" modalities expressing the *"local availability" of information*.

A form of *spatial logic* is needed, capturing notions of "subsystem" (location/part) of a bigger system.

Ontic and Epistemic effect of actions

Non-classical Dynamics

- The non-classical dynamics is reflected in the erosion of the sharp classical distinction between "ontic" and "epistemic" (information-gathering) actions.

- In the quantum world, all information-gathering actions have **ontic side-effects**. *There can be no information change without changing the world.*

- The ontic effects of quantum epistemic actions may be **non-local**. An information-gathering action on one part of a quantum system may affect the ontic state of other, far-away parts of the system.

Adding classical agents: "epistemic" actions

Epistemic Effect

Measurements are now "epistemic" actions: *information-gathering* actions, which convert potential local information into actual information for an agent.

Quantum Holism

- A composed system is entangled iff the information carried by the system is more than the "sum" of all the information carried by its parts.
- In epistemic terms: the "knowledge" of an entangled system is more than the "distributed knowledge" of its components.

Quantum DEL

Quantum Logical Gates

If we also add actions of reversible information processing (*unitaries*=*"quantum gates"*), then

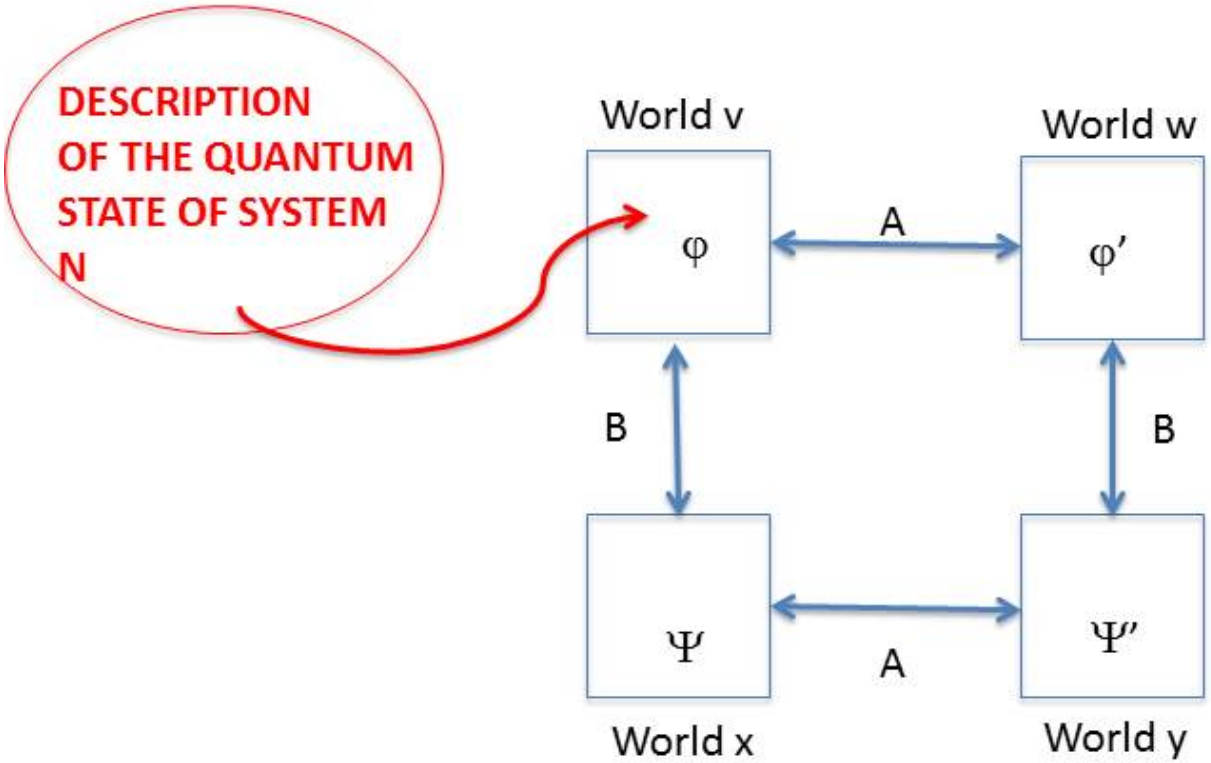
the resulting "Quantum Dynamic Epistemic Logic" can be used to reason about the specific features of quantum information, including *entanglement, Bell states, quantum protocols* (Teleportation, super-dense coding etc).

Formal Part: First Main Ingredients

Main Ingredients (e.g. Teleportation Protocol)

- Family of classical Agents $\mathcal{A}$
- Given (quantum) physical system N
- Classical uncertainty, the knowledge of agents about the quantum system and its properties (but also about what other agents know etc)
- Agents have local control over part/all of a physical system N
- Agents' can perform classical and quantum actions, e.g. from classical communication to quantum measurements. These can affect either or both: the state of system N and the knowledge of the agents .

EPISTEMIC STATE MODEL

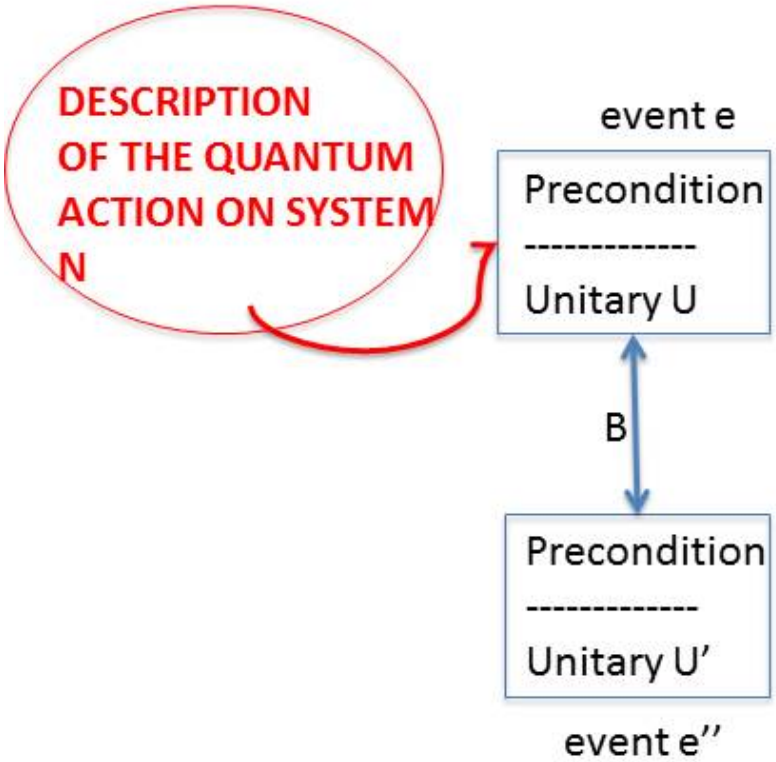


AGENTS : A,B

KNOWLEDGE of A in v = truth in the worlds she considers epistemically possible in v

POTENTIAL KNOWLEDGE of A in v = truth in the worlds she considers epistemically possible + taking into accout the local actions onto the part of system N that's under her control.

EPISTEMIC EVENT MODEL



DEL prescribes via the **product update operation** how we can combine a given state model with an event model (computing the result of an action on a state, keeping track of the epistemic and ontic states).

About Physical Systems

Multi-partite physical systems

Physical system may be compound: composed of subsystems or parts.

This means that the information carried by a physical system can be distributed throughout space:

Information can be **localized**, concentrated at specific spatial "locations" / "sites" / "components" / "situations" of the system.

Distributed Information

So some information is *available* only at some locations, but not others.

Notation for Physical Sytems

System N

Consider a given (finite) set $N = \{1, ..., n\}$ of labels for the **basic "components"** (in a compound system) or "locations".

Subsystems

Besides the basic components of a system, we work also with more complex **subsystems**: *groups of components*, denoted by *sets* $I \subseteq N$ of labels.

State Space

State Space for N

Given a system N, a state space for N is a structure

$$\mathbf{S} = (S_I, p^I_J, \bigotimes)_{J \subseteq I \subseteq N}$$

Where:

- S_I associates to subsystem $I \subseteq N$ some set of "I-local states";
- $p^I_J : S_I \to S_J$ maps I-local states to J-local states (for each $J \subseteq I \subseteq N$);
- for each family $\mathcal{A}$ of mutually disjoint subsystems of N, $\bigotimes_{\mathcal{A}} : \prod_{A \in \mathcal{A}} S_A \to S_{\bigcup \mathcal{A}}$.

Conditions

These are required to satisfy:

- $$p^I_I = id_{S_I},$$
- $$p^J_K \circ p^I_J = p^I_K, \text{ for } K \subseteq J \subseteq I,$$
- $$p^{\bigcup \mathcal{A}}_{\bigcup \mathcal{B}}(\bigotimes_{A \in \mathcal{A}} S_A) = \bigotimes_{B \in \mathcal{B}} S_B, \text{ for } \mathcal{B} \subseteq \mathcal{A}.$$
- NOTATION: $S := S_N$.

Example: Quantum State Spaces

Quantum state space

A **quantum** state space is given by an assignment $(H_i)_{i \in N}$ of Hilbert spaces to each basic component $i \in N$, together with putting:

$$S_I := \{\rho_I : \rho_I \text{ density operator on the tensor product } \bigotimes_{i \in I} H_i\},$$

$$p^I_J(\rho_I) := tr_{I \setminus J}(\rho_I) \ , \ \text{(where } tr \text{ is the partial trace operator)} \ ,$$

$\bigotimes_{A \in \mathcal{A}} \rho_A$ is the standard tensor product of operators $(\rho_A)_{A \in \mathcal{A}}$.

Special Subcase

Qubit spaces: Let each H_i be a two-dimensional Hilbert space.

Actions on (parts of) physical systems

Actions on subsystems $I \subseteq N$

For a subsystem $I \subseteq N$, I-**actions** are maps

$$T_I : S_I \to S_I$$

In the quantum case the actions T_I are required to be **linear**, while in the classical case they are arbitrary maps.

Two important types of actions

- **Quantum** I-**Tests** are actions $P_I?$ induced on the state space S_I by projectors onto some subspace P_I of $H_I = \bigotimes_I H_i$.
- I-**Evolutions** are actions U_I induced on S_I by unitary evolutions $U : H_I \to H_I$.

Global Actions on N

Global actions are the N-actions $T : S \to S$.

I-Local Actions

When a global action T can be called local:

A (global) action $T : S \to S$ is I-**local** if there exists some I-action $T_I : S_I \to S_I$ such that

$$T(s_I \otimes s_{N\setminus I}) = T_I(s_I) \otimes s_{N\setminus I} \quad \text{for all } s_I \in S_I, s_{N\setminus I} \in S_{N\setminus I}.$$

Classical Agents and their Access or Control

Classical Agents

We assume given a fixed set n $\mathcal{A}$ of labels, called **agents**.

Access or Control

An **access map** (or "location map") is a function $l : \mathcal{A} \to \mathcal{P}(\mathcal{N})$ such that

- $A \neq B \;\Rightarrow\; l(A) \cap l(B) = \emptyset$.
- $l(A) \subseteq N$ is called call A**'s location** (or A**'s access**).

This means that, in principle, agent A "has access" or "can control" the qubits in $l(A)$.

Agent's Control is Local

Agent A can perform only "local" actions (local measurements, or local evolutions) on the qubits in $l(A)$.

Local States/Actions for a given Agent

Agents identified with their Location/Access

For a **fixed access map** I, each agent's access $I(A)$ is **fixed**: in this case we can "identify" A with $I(A)$.

Agents' local states and local actions

We can thus refer, for a fixed access map I and a given agent A, to the A-**local state** $s_A := S_{I(A)}$ of (some global state) s, or to A-**actions** ($=I(A)$-actions) etc.

Note

But when agents move, or exchange qubits, the access map changes! So then we cannot maintain the identification between A and $I(A)$.

Multi-agent Actions

I-based multi-agent actions

Given an access map $I : \mathcal{A} \to \mathcal{P}(\mathcal{N})$, a (global) action $T : S \to S$ is **an I-based multi-agent action** if there exist $I(A)$-actions $T_A : S_{I(A)} \to S_{I(A)}$, one for each agent $A \in \mathcal{A}$, such that

$$T(\bigotimes_{A \in \mathcal{A}} s_{I(A)}) = \bigotimes_{A \in \mathcal{A}} T_A(s_{I(A)}).$$

Note

For each agent A, the A-local actions are a special case of multi-agent actions (called **single-agent actions**).

Indistinguishability between Multi-Agent Actions

Modelling Classical Uncertainty of Actions

Two global multi-agent actions are indistinguishable for agent A, if the part she can see/control is the same.

Formally

Given two multi-agent actions $T = \bigotimes_{A \in \mathcal{A}} T_A$ and $T' = \bigotimes_{A \in \mathcal{A}} T'_A$ for the same set of agents, we put

$$T \sim_A T' \quad \text{iff} \quad T_A = T'_A.$$

(Probabilistic Epistemic) State Models

State Model

Given a system N, together with a state space $\mathbf{S}$ and a family $\mathcal{A}$ of agents for N, a **state model** is a tuple $M = (W, \sim, \mu, state, I)$, s.t.

- W is a finite set of "possible worlds";
- $\sim: \mathcal{A} \to \text{Equiv}(W)$ is a map associating to each agent some equivalence relation $\sim_A \subseteq W \times W$, called *A-indistinguishability* (or epistemic accessibility);
- $\mu : W \to [0, 1]$ is a probability distribution over possible worlds, called "the (common) *prior*";
- I is a function that associates to each world $w \in W$ some access map $I_w : \mathcal{A} \to \mathcal{P}(\mathcal{N})$;
- $state : W \to S$ is a map associating to each world $w \in W$ some global state $state(w) \in S$.

State Models Continued

REQUIREMENT: Agents know their location/access

$$w \sim_A w' \Rightarrow I_w(A) = I_{w'}(A).$$

Agents' Epistemic Probabilities

Epistemic Probabilities

Agent A's epistemic probability at a given world w is obtained by conditionalizing μ on w's cell in agent A's information partition $w(A) = \{w' \in W : w \sim_A w'\}$:

$$\mu_A^w(w') = \frac{\mu(w')}{\sum_{v \sim_A w} \mu(v)}, \quad \text{for } w' \sim_A w,$$

and

$$\mu_A^w(w') = 0 \quad \text{for } w' \not\sim_A w.$$

Agents' Epistemic Potential

Local Observations

By performing local observations (i.e. A-local tests), an agent A may obtain more information about the current A-local state

$$state_A(w) := state(w)_A = p^N_{I(A)}(state(w)).$$

So A may be able to better distinguish worlds apart, using her local tests. In the best case, she can fully determine $state_A(w)$.

Epistemic Potential

So A's **potential epistemic indistinguishability** relation $\sim^{\Diamond}_A$ is:

$$w \sim^{\Diamond}_A w' \quad \text{iff} \quad w \sim_A w' \text{ and } state_A(w) = state_A(w').$$

We also put

$$w^{\Diamond}(A) = \{w' \in W : w' \sim^{\Diamond}_A w\}$$

for agent A's **potential-information cell**.

Propositions and Truth

Propositions

A **(epistemic) proposition** is a map $\mathbf{P}$ associating to each model M some set of worlds $\mathbf{P}_M$.

We write

$$w \models_M \mathbf{P} \quad \text{iff} \quad w \in \mathbf{P}_M,$$

and in this case we say that P is **true** at world w in model M (or world w **satisfies** $\mathbf{P}$ in model M).

Epistemic Operators

Our logic has **knowledge operators** K_A and $K_A^{\Diamond}$ for all agents $A \in \mathcal{A}$.

Semantics

Semantics of K_A and $K_A^\Diamond$

The **knowledge**, and **potential knowledge** operators, are defined using the standard *Kripke semantics*: for every proposition **P**, we put

$$w \models K_A\mathbf{P} \text{ iff } v \models \mathbf{P} \text{ for all worlds } v \sim_A w,$$

$$w \models K_A^\Diamond\mathbf{P} \text{ iff } v \models \mathbf{P} \text{ for all worlds } v \sim_A^\Diamond w.$$

In other words:

$$(K_A\mathbf{P})_M := \{w \in W : w(A) \subseteq \mathbf{P}_M\},$$

$$(K_A^\Diamond\mathbf{P})_M := \{w \in W : w^\Diamond(A) \subseteq \mathbf{P}_M\},$$

etc.

Potential Knowledge Operator

Potential Knowledge of agents

The **potential knowledge** operators $K_A^{\Diamond}$, capture (the upper limit of) **what agents could come to know by performing (only) more local observations**.

$K_A^{\Diamond}$ gives only an upper limit: it puts a bound on A's potential knowledge.
In practice (given that local observations may change the system's state), an agent can really actualize only some of this potential.

Probabilistic Operators

Operators for Subjective Probabilities

We could also introduce operators for subjective probabilities:

$$\mu_A(P) \geq r,$$

for every agent A and rational number r;
"agent A assigns at least probability r to proposition P.

More generally, one can follow Halpern et alia and allow in the syntax probabilistic linear inequalities:

$$\sum_{k=1,m} r_i \cdot \mu_A(P_k) \geq r$$

Special Atomic Sentences

A has access to qubit i

We may introduce atomic sentences of the form

$$i \in I(A)$$

"the local I-state of the system is ρ_I "

$$c_I$$

(for some given constants c_I denoting specific local states).

A-Local Propositions

A proposition $\mathbf{P}$ is A-local if, for all models M and all worlds w, w', we have that:

$$w \models \mathbf{P} \text{ and } w \sim_A^{\Diamond} w' \text{ implies } w' \models \mathbf{P}.$$

Examples

For every proposition $\mathbf{P}$, the propositions $K_A\mathbf{P}$ and $K_A^{\Diamond}\mathbf{P}$ are A-local.

Proposition

The following are equivalent:

1. $\mathbf{P}$ is A-local;
2. $\mathbf{P} = K_A^{\Diamond}\mathbf{P}$;
3. $\mathbf{P} = K_A^{\Diamond}\mathbf{Q}$ for some proposition $\mathbf{Q}$.

Special Propositions

Local Propositions

For each I-local state s_I, there exists a corresponding proposition $\mathbf{s}_I$, given by

$$w \models_M \mathbf{s}_I \quad \text{iff} \quad state_I(w) = s_I.$$

More generally, for every subspace P_I of H_I, there exists a corresponding proposition $\mathbf{P}_I$, given by

$$w \models_M \mathbf{P}_I \quad \text{iff} \quad state_I(w) \subseteq P_I.$$

Local Tests

I-local tests can thus be identified with tests $\mathbf{P}_I?$ of propositions of this form.

When $I = A$ is an agent, such propositions are always A-local:

$$\mathbf{P}_A = K_A^{\Diamond} \mathbf{P}_A.$$

(Probabilistic Epistemic) Action Models

Action Model

Given N, $\mathbf{S}$, $\mathcal{A}$ and (an access map) I, an ***I*-action model** is a tuple $\mathcal{M} = (E, \sim, \mu, change, I, J)$

Where:

- E is a set of "possible events";
- $\sim: \mathcal{A} \to \text{Equiv}(E)$ is a map associating to each agent $A \in \mathcal{A}$ some equivalence relation $\sim_A \subseteq E \times E$ on events, called *A-indistinguishability* (or epistemic accessibility);
- $\mu : E \to [0, 1]$ is a probability distribution;
- I and J associate to each event $e \in E$ some access maps I_e, J_e;
- *change* is a map assigning to every event $e \in E$ some type of state change $change(e) = (\Phi_e, Act_e)$, where:

State Changes

$change(e) = (\Phi_e, Act_e)$

- Φ_e is a set of mutually incompatible propositions, called **preconditions** (of event e);
- Act_e is a set of I_e-based multi-agent actions.

REQUIREMENTS

$$e \sim_A e' \;\Rightarrow\; I_e(A) = I_{e'}(A) \wedge J_e(A) = J_{e'}(A),$$

$$T \in Act_e, \rho, \rho' \in P \in \Phi_e \;\Rightarrow\; Tr(T^\dagger T \rho) = Tr(T^\dagger T \rho').$$

Intuition

An event e can only happen if one its preconditions is true:

- Event e may be non-deterministic: one of the actions in Act_e is happening in state ρ with probability $Tr(T^\dagger T \rho)$;
- The probability of an action $T \in Act_e$ happening in a state depends only on the precondition $P \in \Phi_e$ holding in that state;
- Each action $T \in Act_e$ is actually a I_e-multi-agent action: every agent A performs the corresponding local action T_A on the set of qubits $I_e(A)$ to which it has access;
- The agents may lose/acquire/exchange qubits (without necessarily knowing that), change locations etc, so that after event e, agent A's new location or set of accessible qubits is $J_e(A)$.

Agents' Epistemic Probabilities for Events

Epistemic Probabilities

Agent A's epistemic probability at a given event e is obtained by conditionalizing μ on e's cell in agent A's information partition $e(A) = \{e' : e' \sim_A e\}$:

$$\mu_A^e(e') = \frac{\mu(e')}{\sum_{f \sim_A e} \mu(f)}, \quad \text{for } e' \sim_A e,$$

and

$$\mu_A^e(e') = 0 \quad \text{for } e' \not\sim_A e.$$

Example: local measurements in a commonly-known basis

Event Model for agent A's local observation:

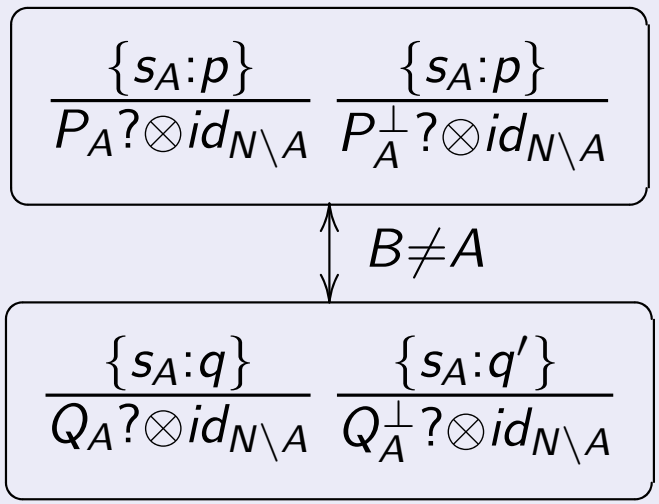
$$\frac{\{s_A{:}p\}}{P_A?\otimes id_{N\setminus A}} \quad \frac{\{s_A{:}p'\}}{P_A^{\perp}?\otimes id_{N\setminus A}}$$

where $\mu = 1$, loops are implicit for all agents and

$$p = Tr(P_A?s_A)$$

$$p' = Tr(P_A^{\perp}?s_A)$$

Example: measurements in one of a set of possible bases



where (say) $\mu = \frac{1}{2}$ for both events, p, p' as before and

$$q = Tr(Q_A?s_A)$$

$$q' = Tr(Q_A^{\perp}?s_A)$$

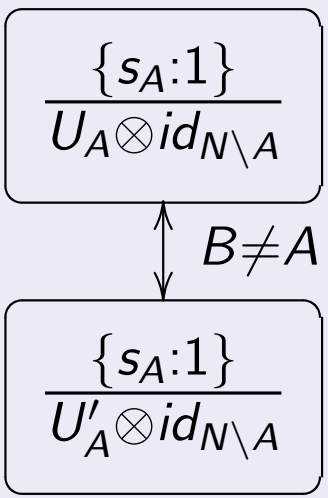
Example: public local quantum operations

Public local quantum opertions

$$\frac{\{s_A{:}1\}}{U_A \otimes id_{N\setminus A}}$$

where $\mu = 1$.

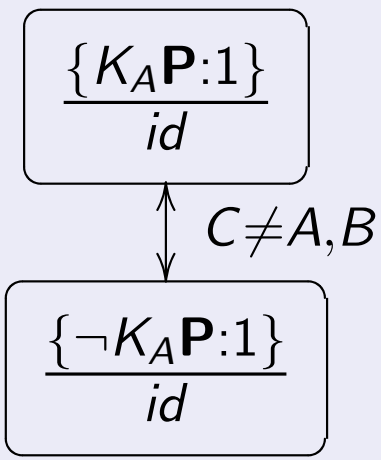
Example: performing one of a set of possible operations



where say $\mu = \frac{1}{2}$ for both events.

Example: classical communication

It is common knowledge that A privately communicates to B whether or not she knows $\mathbf{P}$, without changing the system's quantum state.



where say $\mu = \frac{1}{2}$ for both events.

Example: (classical) public announcement

A publicly announces (that she knows) that P.

$$\frac{\{K_A\mathbf{P}{:}1\}}{id}$$

with $\mu = 1$.

The effect of a Quantum Action on a Quantum State

Occurence Probability of a Quantum Action

We define a map $pr_e : \Phi_e \to (Act_e \to [o, 1])$, associating to each precondition $\mathbf{P} \in \Phi_e$ a probability distribution over possible actions in Act_e,

It gives for each action $T \in Act_e$ the (conditional) probability $pr(T|\mathbf{P})$ that T will occur in a state satisfying $\mathbf{P}$.

pr_e is given by **Born's rule**:

$$pr_e(T|P) = Tr(T^\dagger T \rho) \quad \text{for } \rho \in P \in \Phi_e, T \in Act_e.$$

Update Product: Compute the updated state model after an action

Following Baltag-Moss-Solecki-98

Given **a state model** $M = (W, \sim, \mu, state, I)$ and **an action model** $\mathcal{M} = (E, \sim, \mu, change, I, J)$, the **new state model after the action** is

$$M \bigodot \mathcal{M} = (W \bigodot E, \sim, \mu, state, I), \text{ where}$$

- $W \bigodot E = \{(w, e, T) : w \in W, e \in E, T \in Act_e$ such that $w \models_M \mathbf{P}$, $I_w = I_e$ and $pr(T|\mathbf{P}) > 0\}$,
- $(w, e, T) \sim_A (w', e', T')$ iff $w \sim_A w'$, $e \sim_A e'$ and $T \sim_A T'$;
- $\mu(w, e) = \mu(w) \cdot \mu(e) \cdot pr_e(T|w)$, where we put
  - $pr_e(T|w) = pr_e(T|\mathbf{P}_w)$, if there exists $\mathbf{P}_w \in \Phi_e$ s.t. $w \models_M \mathbf{P}$,
  - $pr_e(T|w) = 0$, otherwise.

Product Update Continued

- $state(w, e, T) = T(state(w)) = \frac{Tstate(w)T^{\dagger}}{Tr(T^{\dagger}Tstate(w)}$;
- $I_{(w,e,T)} = J_e$.

Dynamic Epistemic Logic

Syntax: add dynamic operators

$$[e, T]P, \quad [e]P$$

Semantics: use the updated model

$$w \models_M [e, T]\varphi \text{ iff } (w, e, T) \models_{M \cdot \mathcal{M}} \varphi,$$

$$w \models_M [e]\varphi \text{ iff } w \models_M [e, T]\varphi \text{ for all } T \in Act_e.$$

We obtain "Reduction Axioms", that allows us to pre-compute future informational changes of the form

$$[e]K_A P$$

or

$$[e] \sum_{k=1,m} r_i \cdot \mu_A(P_k) \geq r$$

Applications: Teleportation

Quantum teleportation

a technique that makes it possible to teleport "the state" of a quantum system. We are working in a 3-qubit space $S_1 \otimes S_2 \otimes S_3$.

Two agents, Alice and Bob who are separated in space.

Alice controls qubit 1, in unknown local state $q_1 \in S_1$. In addition, she also has an "entangled EPR pair", i.e. a pair of qubits $2, 3$ in the Bell state $\beta^{00}_{2,3} \in S_2 \otimes S_3$. Alice gives the entangled qubit 3 to

Bob (send it to him, and somehow she's sure he receives it).

Teleportation, continued

The program

Alice wants to "teleport" the unknown qubit state to Bob, i.e. she will perform **a program** that will output a state satisfying $id_{13}(q_1)$.

Protocol

She first entangles qubit 1 with her part (qubit 2) of the EPR pair. She does this by performing a $CNOT_{1,2}$ gate on the two qubits and then a Hadamard transformation H_1 on the first component.

Teleportation continued

Next, Alice measures her qubits in the standard basis, destroying the entanglement. As a result, Bob's qubit (3) will collapse to a state q_3.

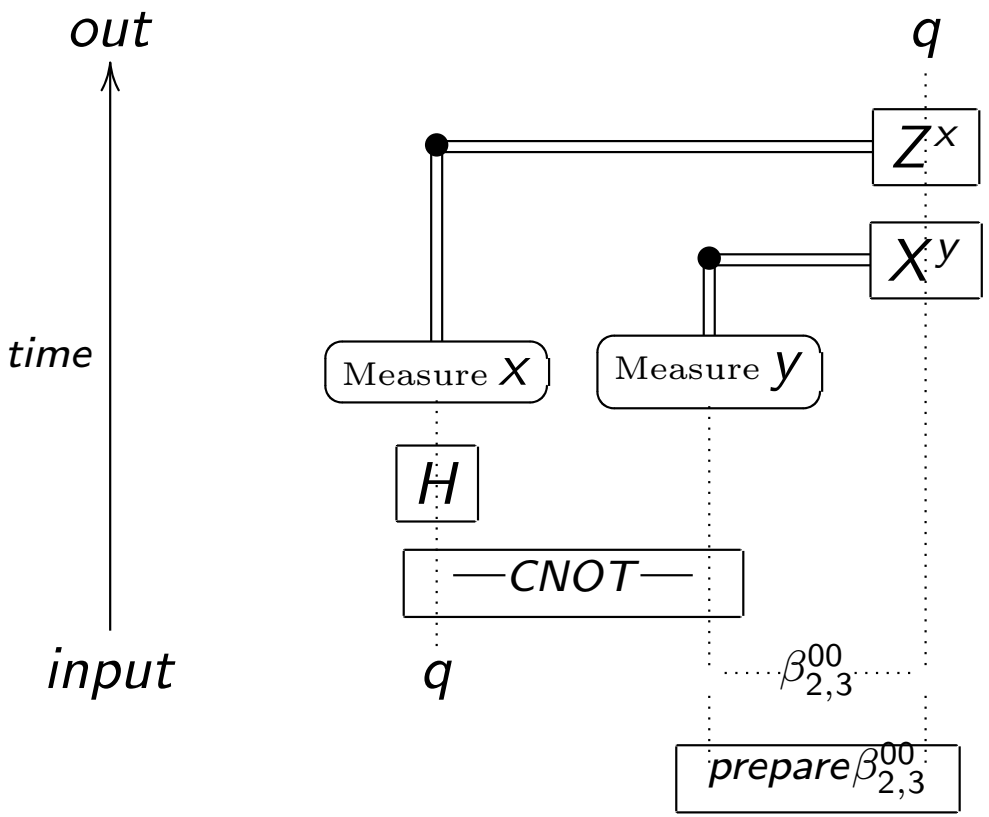
Moreover, *the result that Alice obtains from the two measurements indicate the actions that Bob has to perform in order to transfer his qubit from its state q_3 into the same state as Alice's original qubit*, i.e. the state $id_{13}(q_1)$ (corresponding to the qubit Alice had before the protocol).

Communication part

It is enough for Alice to communicate to Bob the result of her measurement, i.e. send him two classical bits $(x, y) \in \{0, 1\} \times \{0, 1\}$, encoding the result x_1 of the first measurement and the result y_2 of the second measurement.

This tells Bob that he will have to apply y times the X-gate followed by x times the Z gate, if he wants to force his qubit q_3 into the state $id_{13}(\varphi_1)$.

Actual physics of teleportation:



Encoding

System:

$$N = \{1, 2, 3\}$$

Agents' Control:

$$\text{Initially: } I(A) = \{1, 2, 3\}, I(B) = \emptyset.$$

State Space:

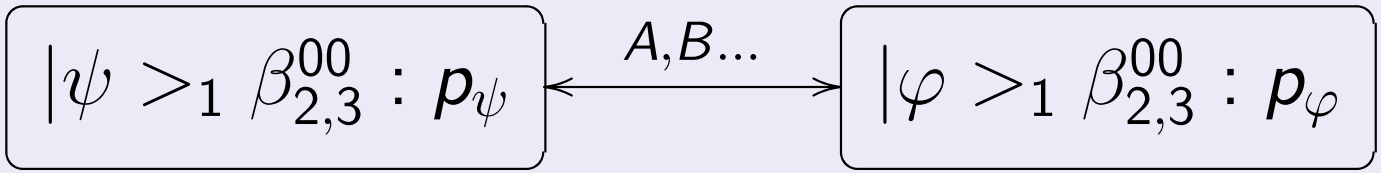
S is the state space of $H_2 \otimes H_2 \otimes H_2$.

Initial State Model

Assumptions made explicit

Suppose that it is **common knowledge** that A has entangled qubits $2, 3$ in Bell state β^{00}, but that **nobody knows the state of her qubit** 1. Moreover, there is some common prior probabilistic belief μ about the state of qubit 1.

Initial state model $(W, \sim, state)$:



with

$$\mu(|\psi >_1 \beta^{00}_{2,3}) = p_\psi,$$

$$I_w(A) = \{1, 2, 3\}, I_w(B) = \emptyset \text{ in ALL worlds } w,$$

etc.

Relaxing the Common Knowledge Assumptions

Changing the assumptions

What if everything is as above, except that now (it is common knowledge that) B **doesn't know whether** A **knows her qubit** 1 **or not**.

EXERCISE: Draw The Picture!

Further Variations Possible

Everything is as in the first model, but now there is an **intruder** E who doesn't know whether A knows her qubit 1 or not (but she knows that B doesn't know the state of qubit 1)?

AN ACTION HAPPENS: Alice gives qubit 3 to Bob

Assumptions

Let's back to the first model: assume common knowledge of ignorance of (state of) qubit 1.
Moreover, assume now that **the Teleportation Protocol is also common knowledge**.

Event e

The event e of Alice giving qubit 3 to Bob is thus a *public* one:

$$\frac{\{\top{:}1\}}{id_{1,2,3}}$$

with $\mu = 1$, $I_e = I_w$, $J_e(A) = \{1, 2\}$, $J_e(B) = \{3\}$.
This changes the model only by changing its access map to

$$I'_w := J_e.$$

Alice performs her unitary operations

Since the protocol is common knowledge, Alice's local unitary operations are also **public**:

The action model for A's unitary operations

$$\frac{\{s_A{:}1\}}{(CNOT_{1,2};H_2)\otimes id_3}$$

where $\mu = 1$.

The new state model

The state model after this is computed using update product:

$$\sum_{i,j} |ij\psi^{(i,j)} >_{1,2,3} : p_\psi \xleftrightarrow{A,B\ldots} \sum_{i,j} |ij\varphi^{(i,j)} >_{1,2,3} : p_\varphi$$

with p_ψ, p_φ as before and $\psi^{(i,j)} = X^j(Z^i(\psi)$ (and similarly for φ etc).

Alice's measurement of first qubit in the canonical basis

Alice's first measurement (of qubit 1 in the canonical basis) is a local measurement in a commonly known basis.
(This is because we assumed common knowledge of the protocol)

Action Model of A's first measurement

$$\frac{\{s_A : \frac{1}{2}\}}{|0>_1? \otimes id_{2,3}} \quad \frac{\{s_A : \frac{1}{2}\}}{|1>_1? \otimes id_{2,3}}$$

where $\mu = 1$.

Alice's measurement of second qubit

The same applies to A's measurement of the second qubit:

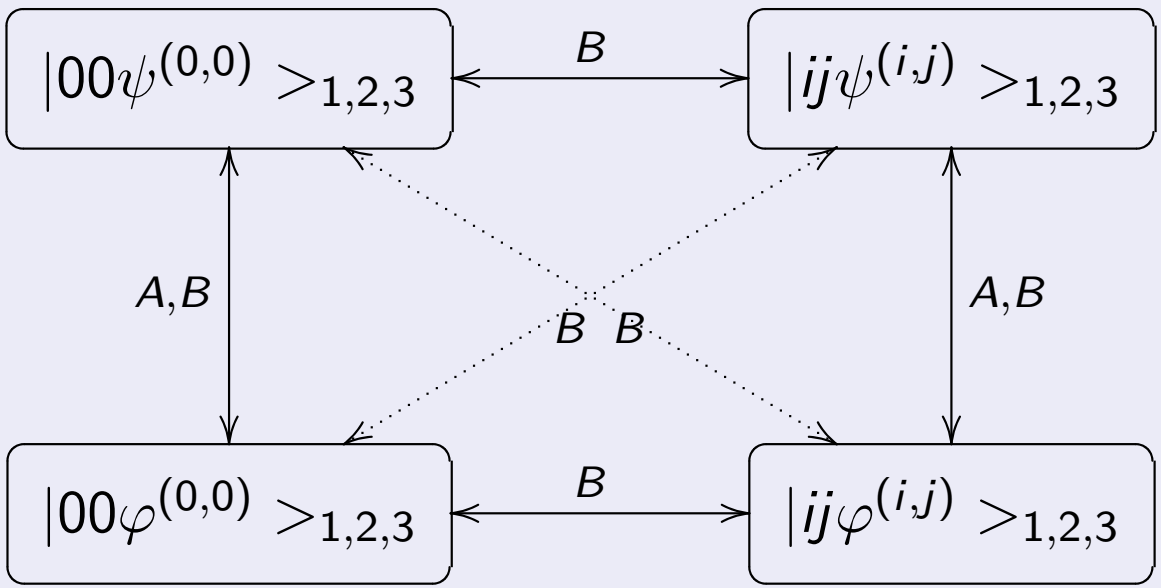
Action model of A's second measurement

$$\frac{\{s_A : \frac{1}{2}\}}{id_1 \otimes |0>_2? \otimes id_3} \quad \frac{\{s_A : \frac{1}{2}\}}{id_1 \otimes |1>_2? \otimes id_3}$$

where $\mu = 1$.

The Resulting State Model

The resulting state model after these measurements is:



Final Step: Communication and Final State Model

Neglecting intruders, we can assume that this is a **public communication** of $K_A|ij>_{1,2}$, for the **true** $i, j \in \{0, 1\}$.
The resulting model (on the right) is obtained by disconnecting all arrows except for the vertical ones: