ABSTRACTS

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Fluids and Variational Methods

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INVITED TALKS

Variational problems in geometry of random tilings

Kari Astala
Aalto

Scaling limits of discrete random structures in two dimensions often posses some conformal invariance properties. Random tilings are typical examples, where understanding the geometry of their scaling limits leads to a free boundary problem with a gradient constraint.

However, now the energy functional is partially singular, so that the usual highly developed methods for free boundary problems are not available. In this presentation, on joint work with E.Duse, I.Prause and X.Zhong, I explain how with a combination of quite different tools, from variational methods to the Beltrami differential equation, one can understand the geometry of ordered and disordered (or frozen and liquid) regions of scaling limits of random tilings.

From the d'Alembert paradox to the 1984 Kato criteria via the 1941 1/3 Kolmogorov law and the 1949 Onsager conjecture.

Claude Bardos
Emeritus-Professor, Laboratory Jacques Louis Lions, Université Paris−Diderot

Several of my recent contributions, with Marie Farge, Edriss Titi, Emile Wiedemann, Piotr and Agneska Gwiadza, were motivated by the following issues:

The role of boundary effect in mathematical theory of fluids mechanic and the similarity, in presence of these effects, of the weak convergence in the zero viscosity limit and the statistical theory of turbulence. As a consequence. I will recall the Onsager conjecture and compare it to the issue of anomalous energy dissipation.

Then I will give a proof of the local conservation of energy under convenient hypothesis in a domain with boundary and give supplementary condition that imply the global conservation of energy in a domain with boundary and the absence of anomalous energy dissipation in the zero viscosity limit of solutions of the Navier-Stokes equation in the presence of no slip boundary condition.

Eventually the above results are compared with several forms of a basic theorem of Kato in the presence of a Lipschitz solution of the Euler equations and one may insist on the fact that in such case the the absence of anomalous energy dissipation is **equivalent** to the persistence of regularity in the zero viscosity limit. Eventually this remark contributes to the resolution of the d'Alembert Paradox.

Lipschitz bounds and non-uniform ellipticity

Lisa Beck
Augsburg

In this talk we consider a large class of non-uniformly elliptic variational problems and discuss optimal conditions guaranteeing the local Lipschitz regularity of solutions in terms of the regularity of the data. The analysis covers the main model cases of variational integrals of anisotropic growth, but also of fast growth of exponential type investigated in recent years. The regularity criteria are established by potential theoretic arguments, involve natural limiting function spaces on the data, and reproduce, in this very general context, the classical and optimal ones known in the linear case for the Poisson equation. The results presented in this talk are part of a joined project with Giuseppe Mingione (Parma).

Various formulations and approximations of the Muskat model

Yann Brenier, CNRS
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Abstract

The Muskat model, which describes the filtration of an incompressible fluid in a porous medium, has played a crucial role in both optimal transport and convex integration theories. I will first recall how a very simple and computationally effective time discrete version of the Muskat model can be designed in terms of polar factorization of maps. Then I will discuss various possible formulations (multiphasic, dissipative...) of this model.

Wild solutions of the Navier-Stokes equations may be smooth for a.e. time

Maria Colombo
EPFL Lausanne

In a recent result, Buckmaster and Vicol proved non-uniqueness of weak solutions to the Navier-Stokes equations which have bounded kinetic energy and integrable vorticity. We discuss the existence of such solutions, which in addition are regular outside a set of times of dimension less than 1.

Remarks on Euler and Navier-Stokes equations

Peter Constantin
Princeton

I will describe briefly basic questions of the area. Then I will discuss some recent results: 1) smooth multiscale solutions of three dimensional incompressible Euler equations with compactly supported velocity and 2) conditions away from the boundary for the zero viscosity limit to be given by possibly dissipative solutions of Euler equations.

Fluid-Squeezing singularities for the incompressible Euler equations

Diego Cordoba Gazolaz
UAM Madrid

I will discuss a new result on the existence of a stationary solution with a fluid-squeezing singularity for the two-fluid incompressible Euler equations. The proof is based on a new set of estimates that permits us to analyze a fluid squeezed by a self-intersecting interface. We will exploit these estimates in the dynamical case and show a local existence result starting from a fluid-squeezing singularity.

Rigidity and flexibility of isometric embeddings

Camillo De Lellis
Princeton

Consider a smooth connected closed two-dimensional Riemannian manifold Σ with positive Gauss curvature. If u is a C^2 isometric embedding of Σ, then $u(\Sigma)$ is convex. In the fifties Nash and Kuiper showed, astonishingly, that this is not necessarily true when the map is C^1. It is expected that the threshold at which isometric embeddings "change nature" is the $\frac{1}{2}$-Hoelder continuity of their derivatives, a conjecture which shares a striking similarity with a (recently solved) problem in the theory of fully developed turbulence. In my talk I will review several plausible reasons for the threshold and a very recent work, joint with Dominik Inauen, which indeed shows a suitably weakened form of the conjecture.

Generic regularity in obstacle problems

Alessio Figalli
ETH Zurich

The so-called Stefan problem describes the temperature distribution in a homogeneous medium undergoing a phase change, for example ice melting to water. An important goal is to describe the structure of the interface separating the two phases. In its stationary version, the Stefan problem can be reduced to the classical obstacle problem, which consists in finding the equilibrium position of an elastic membrane whose boundary is held fixed and which is constrained to lie above a given obstacle. The aim of this talk is to discuss sone recent developments on the generic regularity of the free boundary in both problems.

Non Linear Beltrami Equation

Daniel Faraco
UAM Madrid

The nonlinear Beltrami equation parametrizes all planar elliptic systems and it has shown to be an important tool in various problems running from quasiconvexity to random surfaces. It is also intimately connected to quasilinear elliptic equations. We will present various surprising results concerning uniqueness and regularity. The structure of solutions will be also described. These are joint works with Kari Astala, Aleksis Koski, Jarmo Jääskeläinen and L.Székelyhidi Jr.

Recent results on singular limits for Vlasov-Poisson

Mikaela Iacobelli
ETH Zurich

The Vlasov-Poisson system is a kinetic equation that models collisionless plasma. A plasma has a characteristic scale called the Debye length, which is typically much shorter than the scale of observation. In this case the plasma is called quasineutral. This motivates studying the limit in which the ratio between the Debye length and the observation scale tends to zero. Under this scaling, the formal limit of the Vlasov-Poisson system is the Kinetic Isothermal Euler system. The Vlasov-Poisson system itself can formally be derived as the limit of a system of ODEs describing the dynamics of a system of N interacting particles, as the number of particles approaches infinity. The rigorous justification of this mean field limit remains a fundamental open problem.

In this talk we present how the mean field and quasineutral limits can be combined to derive the Kinetic Isothermal Euler system from a regularised particle model.

Electrodiffusion of ions in fluids

Mihaela Ignatova
Temple University

The electrodiffusion of ions in fluids is governed by the Nernst-Planck-Navier-Stokes system. We prove global existence and stability results for large data, in two dimensions, with Dirichlet boundary conditions for the Navier-Stokes and Poisson equations, and blocking (vanishing normal flux) or selective (Dirichlet) boundary conditions for the ionic concentrations, for arbitrary Reynolds number, voltages, ionic valences, and species diffusivities. The proofs employ a new remarkable structure resulting in the decay of the sum of relative entropies of the ionic concentrations and the kinetic energy of the fluid. This is joint work with Peter Constantin.

Brunn-Minkowski and entropy inequalities: from Riemannian to sub-Riemannian

Alexandru Kristaly
Babes-Bolyai University, Cluj-Napoca and Obuda University

We first recall some geometric and functional inequalities on Riemannian manifolds a la Cordero-Erausquin, McCann and Schmuckenschlager which stood as starting points in the theory of Lott-Sturm-Villani. In particular, we characterize the equality cases in distorted Brunn-Minkowski inequalities on Riemannian manifolds with Ricci curvature bounded from below. We then present certain Brunn-Minkowski and entropy inequalities in the sub-Riemannian Heisenberg/Carnot groups by using optimal mass transportation, pointing out deep differences between the Riemannian and sub-Riemannian settings. Talk based on joint papers with Zoltan M. Balogh (Bern).

Gårding inequalities and their impact on regularity and uniqueness

Jan Kristensen
University of Oxford

Minimizers of strongly quasiconvex variational integrals need not be regular nor unique. However, if a suitable Gårding type inequality is assumed for the variational integral, then both regularity and uniqueness of minimizers can be restored under natural smallness conditions on the data. In turn, the Gårding inequality turns out to always hold under an a priori C^1 regularity hypothesis on the minimizer, while its validity is not known in the general case. In this talk, we discuss these issues and how they are naturally connected to convexity of the variational integral on the underlying Dirichlet classes. We also discuss the problem of regularity for minimizers in the two-dimensional case where stronger results can be established.

Analysis of Oceanic and Tropical Atmospheric with Moisture Models: Global Regularity, Finite-time Blowup and Singular Limit Behavior

Edriss S. Titi
University of Cambridge
Texas A&M University
and
Weizmann Institute of Science

In this talk, I will present some recent results concerning global regularity of certain geophysical models. This will include the three-dimensional primitive equations with various anisotropic viscosity and turbulence mixing diffusion, and certain tropical atmospheric models with moisture. Moreover, in the non-viscous (inviscid) case it can be shown that there is a one-parameter family of initial data for which the corresponding smooth solutions of the primitive equations develop finite-time singularities (blowup).

Capitalizing on the above results, we can provide rigorous justification of the derivation of the Primitive Equations of planetary scale oceanic dynamics from the three-dimensional Navier-Stokes equations, for vanishing small values of the aspect ratio of the depth to horizontal width. Specifically, we can show that the Navier-Stokes equations, after being scaled appropriately by the small aspect ratio parameter of the physical domain, converge strongly to the primitive equations, globally and uniformly in time, and that the convergence rate is of the same order as the aspect ratio parameter. Furthermore, I will also consider the singular limit behavior of a tropical atmospheric model with moisture, as $\varepsilon \to 0$, where $\varepsilon > 0$ is a moisture phase transition small convective adjustment relaxation time parameter.

Regular and singular solutions of certain 1d model equations

Vladimir Šverák
University of Minnesota

We will discuss the De Gregorio model and a few other model equations.

CONTRIBUTED TALKS

Energy conservation and Lagrangian solutions to the 2D Euler equations obtained via the vortex method

Gennaro Ciampa (Gran Sasso Science Institute)

The motion of a two-dimensional incompressible fluid is governed by the 2D Euler equations

$$\begin{cases} \partial_t v + (v \cdot \nabla)\, v + \nabla p = 0, \\ \operatorname{div} v = 0, \\ v(0, \cdot) = v_0, \end{cases} \tag{1}$$

where $v : [0,T] \times \mathbb{R}^2 \to \mathbb{R}^2$ is the velocity field, $p : [0,T] \times \mathbb{R}^2 \to \mathbb{R}$ is the scalar pressure and $v_0 : \mathbb{R}^2 \to \mathbb{R}^2$ is a given divergence-free initial velocity. Smooth solutions, whose global existence is well-known, enjoy two very natural properties: the first one is that they are *Lagrangian*, namely the vorticity is advected by the flow of the velocity; the second property is that smooth solutions conserve the *kinetic energy*. When we consider solutions in weaker classes, precisely when the initial vorticity is in L^p with $1 \leq p < \infty$, the existence of Lagrangian solutions and the conservation of the energy depend in general on the approximation scheme, in particular when the initial vorticity has low integrability. In this talk we prove that solutions obtained via the vortex-blob method are Lagrangian when $p \geq 1$, and that conserve the kinetic energy if $p > 1$.
Based on a joint work with G. Crippa (University of Basel) and S. Spirito (University of L'Aquila).

The Biot-Savart operator of a bounded domain

María Ángeles García-Ferrero (Max Planck Institute, Leipzig)

The Biot-Savart operator plays a key role in fluid mechanics mapping the vorticity of a fluid into its associated velocity field. I will explain the construction of the analog of the BiotSavart integral for bounded domains. Specifically, I will show that the velocity field of an incompressible fluid with tangential boundary conditions on a bounded domain can be written in terms of its vorticity using an integral kernel that has an inverse-square singularity on the diagonal. This is a joint work with A. Enciso and D. Peralta-Salas.

Morrey's Problem and a Conjecture of Šverák

Andrá Guerra (University of Oxford)

Quasiconvexity and rank-one convexity play a fundamental role in the vectorial Calculus of Variations; Morrey's problem is to decide whether these notions coincide. In this talk we will see that to solve this problem it is sufficient to check whether all extremal rank-one convex functions are quasiconvex. We will also identify a class of such extremal functions, thus proving a conjecture made by Šverák in 1992.

Pushing Forward the Theory of Well-Posedness for Systems of Conservation Laws Verifying a Single Entropy Condition

Sam G. Krupa (University of Texas)

For hyperbolic systems of conservation laws in one space dimension, the best theory of well-posedness is restricted to solutions with small total variation (Bressan et al. 2000). Looking to expand on this theory, we push in new directions. One key difficulty is that for many systems of conservation laws, only one nontrivial entropy exists. In 2017, in joint work with A. Vasseur, we proved uniqueness for the solutions to the scalar conservation laws which verify only a single entropy condition. Our result was the first result in this direction which worked directly on the conservation law. Further, our method was based on the theory of shifts and a-contraction developed by Vasseur and his team. These theories are general theories and apply also to the systems case, leading us to hope the framework we built for the scalar conservation laws will apply to systems. In this talk, I review the current progress on using the theory of shifts and a-contraction to push forward the theory of well-posedness for systems of conservation laws in one space dimension. This is joint work with A. Vasseur.

Taylor's conjecture on magnetic helicity conservation
Sauli Lindberg (University of Helsinki)

In 3D ideal (i.e. inviscid, nonresistive) magnetohydrodynamics, magnetic helicity is one of the conserved integral quantities. Simulations and theoretical results indicate that magnetic helicity is, in fact, a much more robust conserved quantity than total energy (the sum of kinetic and magnetic energies), and Taylor conjectured in 1974 that magnetic helicity is approximately conserved at very small resistivities. The conjecture plays a fundamental part in Taylor relaxation theory which has been rather successful in predicting relaxed states of various laboratory plasmas. In the talk I will discuss the physical basis of Taylors conjecture.

Mathematically, Taylor's conjecture says that magnetic helicity is conserved in the ideal limit. I will discuss the recent proof, in joint work with Daniel Faraco, of the mathematical version. In multiply connected domains, magnetic helicity is not gauge invariant and, as such, not conserved without extra assumptions on the vector potential. However, many laboratory plasma configurations are multiply connected, and I present a natural condition on vector potentials under which magnetic helicity is conserved there as well.

Calculus of variations in the anisotropic setting
Adam Prosinski (University of Oxford)

In this talk we will review some recent results concerning existence and regularity of minimisers of anisotropic variational problems. The anisotropy that we have in mind concerns different orders of derivation in different directions, thus we work with differential operators that need not be homogeneous in the usual sense. We will explain how to obtain existence and regularity statements for minimisers of such problems using the usual tools of calculus of variations (such as quasiconvexity) combined with the structural theory of anisotropic Sobolev spaces and regularity theory for linear quasielliptic equations. This talk is based on my on-going collaboration with my doctoral advisor Prof. Jan Kristensen.

On the regularity for non-uniformly elliptic equations

Matthias Schäffner (University of Leipzig)

I will discuss regularity properties for solutions of linear second order non-uniformly elliptic equations in divergence form. Assuming certain integrability conditions on the coefficient field, we obtain local boundedness and validity of Harnack inequality. The assumed integrability assumptions are essentially sharp and improve upon some classical results in the literature. If time permits I will discuss applications to the regularity of minimizers of integral functionals with 'non-standard' growth conditions.

Lower Semi-continuity of Positive Symmetric Divergence-free Matrix valued Functions

Jack Skipper

In the recent work of Denis Serre (2017) he proved a Jensen's inequality for positive definite symmetric divergence-free matrix valued functions (DPT's), a framework for many compressible fluid systems. He conjectured to us that from this one can show lower semi-continuity of the relating functional. We will show how this relates to generating a homogeneous young measure from a sequence DPT's and that we can generate this homogeneous young measure in certain cases.

Gradient flows of inextensible curves and membranes

Dmitry Vorotnikov (University of Coimbra)

Inextensible curves and membranes have intrinsic similarities with incompressible fluids. The motion of these objects is determined by geodesics (or Newton equations) on certain spaces of volume-preserving immersions (for fluids this goes back to Vladimir Arnold, if not Euler himself). In this talk, I will speak about well-posedness and long-time asymtoptics of some gradient flows on these spaces: the gradient flow of the potential energy and the "uniformly compressing mean curvature flow".