Fractal models in biology

Wolfgang Wertz

Technische Universität Wien

At the start we discuss why classical geometry frequently fails in providing appropriate mathematical models for biological objects and processes. The importance of a high surface/volume ratio (which approximately equals the surface/mass ratio) for a sufficient supply of an organism with nutrient media, energy etc., necessary for vital functions suggests the use of fractal geometry in biological models. This argument is illustrated by theoretical considerations about fractal curves in the plane as well as some phenomena occurring in nature, such as diffusion limited aggregation (DLA) and Brownian motion.

The connection of different notions of dimension is discussed, and the advantages and the drawbacks of the use of outer box counting dimension are treated, in particular, some aspects of empirical estimation of box counting dimension by log-log-diagrams are dealt with.

Such models can be used for investigation of tumours and of nuclei of tumour cells. Empirical data of nuclei of different kinds of tumours show considerably higher outer box counting dimension of sections of injured cells kernels compared to sound ones. Moreover, dimension is increasing with the progression of cancer.

When carefully using a theorem concerning Hausdorff dimension of sections with linear manifolds, one can infer the dimension of three-dimensional objects from the dimension of plain sections. A corresponding result for Box counting dimension does not seem to have been proved yet.