A stochastic calculus proof of the CLT for the L^2 modulus of continuity of Brownian local time

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Using stochastic calculus, we give a relatively short proof of the Central Limit Theorem

$$\frac{\int (L_t^{x+h} - L_t^x)^2 \, dx - 4ht}{h^{3/2}} \stackrel{\mathcal{L}}{\Longrightarrow} c \left(\int (L_t^x)^2 \, dx \right)^{1/2} \eta$$

as $h \to 0$ for Brownian local time L_t^x. Here η is an independent normal random variable with mean zero and variance one. Our proof makes use of differentiability properties of the double and triple renormalized self-intersection local time in the spatial variable.