The spacings ratio empirical process

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Let $X_{1,n} \leq \ldots \leq X_{n,n}$ and $Y_{1,n} \leq \ldots \leq Y_{n,n}$ denote the order statistics based upon two independent random samples $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$ of random variables with densities f_X and f_Y on $\mathbb{R}$. We assume that $n \geq 2$, and that f_X and f_Y are positive and continuous on an interval $(a, b) \subseteq \overline{\mathbb{R}}$, and equal to 0 on $\overline{\mathbb{R}} - [a, b]$. We consider the spacings defined by $S_{i;X} = X_{i+1,n} - X_{i,n}$ and $S_{i;Y} = Y_{i+1,n} - Y_{i,n}$, for $1 \leq i \leq N := n - 1$. The spacings ratio empirical process is then defined by $\gamma_N(t) = N^{1/2}(G_N(t) - t)$ for $0 \leq t \leq 1$, where $G_N(t)$ is the empirical distribution function based upon $\{R_{i,N} := S_{i;X}/(S_{i;X} + S_{i;Y}) : 1 \leq i \leq N\}$. The purpose of this talk is to show that, when $f_X = f_Y = f$, the spacing ratio process γ_N converges to a limiting Gaussian process, whose structure is explicitly described in terms of f. Applications to tests of homogeneity are provided, together with extensions of these results to the case where sample sizes are unequal. This research is joint work with Gérard Derzko.

Key words: Empirical processes; Gaussian processes; Invariance principles; Order statistics; Spacings; Tests of homogeneity; Weak laws.