

Density of rational points on K3 surfaces and their symmetric products

Brendan Hassett (Rice University, Houston)

Let S be a projective variety defined over a number field. Rational points on S are said to be potentially dense if, over some number field, the rational points of S are not contained in any proper subvariety. For example, it is not hard to show that rational points are potentially dense for unirational varieties, elliptic curves, and abelian varieties.

The case of K3 surfaces is far more subtle. Bogomolov, Harris, and Tschinkel have shown that rational points are potentially dense for many special classes of K3 surfaces, but no general results are known. However, general results have been obtained after changing the question slightly. Let $Sym^n(S)$ denote the n -th symmetric product of S , which admits a natural desingularization with trivial canonical bundle. One therefore expects the behavior of the rational points on S and $Sym^n(S)$ to be strongly similar, a marked departure from the situation for curves. Work of Tschinkel and Hassett shows that there exist values of n , explicitly computable from projective invariants of S , such that $Sym^n(S)$ has potentially dense rational points.

This course will give an overview of the geometric techniques used to prove density of rational points for varieties with trivial canonical bundle. Detailed proofs will be given in special cases, e.g., quartic surfaces containing a line. One common technique for producing rational points is to find fibrations by subvarieties, like elliptic curves or abelian varieties, which obviously admit many rational points. The geometry of such fibrations is an extremely active topic of current research (cf. the work of Yau-Zaslow and D. Matsushita), and it is of great interest to find simple criteria for their existence.