**Homework 1**
due October 14, 2022.

1. Prove that if for two finite simple graphs $F$ and $G$ we have $F \to G$ (that is, there exists a graph homomorphism from $F$ to $G$) then

$$C_{\mathrm{OR}}(F) \leq C_{\mathrm{OR}}(G).$$

[3 points]

2. Show without using the Strong Perfect Graph Theorem that a graph $G$ has the property that $\chi_f(H) = \omega(H)$ for every induced subgraph $H$ of $G$ if and only if $G$ is perfect.

[4 points]

3. Characterize all classes $\mathcal{C}$ of finite simple graphs that have both of the following properties:

(i) $\mathcal{C}$ is closed under the OR-product, that is

$$F, G \in \mathcal{C} \Rightarrow F \cdot G \in \mathcal{C}$$

and

(ii) $\mathcal{C}$ is closed under taking induced subgraphs, that is

$$G \in \mathcal{C}, G' \subseteq_{\mathrm{ind}} G \Rightarrow G' \in \mathcal{C}.$$

[5 points]