Maximal subgroups of free idempotent generated semigroups

ABSTRACT

The study of idempotents is at the heart of semigroup theory. In 1966 Howie showed that every non-bijective map of a finite set can be written as a product of idempotents, a result that motivated many analogues. If S is a semigroup and $E(S)$ the set of idempotents of S, then $E(S)$ is a partial algebra which may be axiomatised as a *biordered set*. The seminal work of Nambooripad in the 1970s led to a deep understanding of the correspondence between regular semigroups and biordered sets.

Given any biordered set E, the *free idempotent generated semigroup* $IG(E)$ *on* E has biordered set of idempotents E, is generated by E, and maps onto any semigroup with these properties. For 20 years or more it was conjectured that any maximal subgroup of $\mathrm{IG}(E)$ must be free. A counterexample was produced by Brittenham, Margolis and Meakin in 2009. This was swiftly followed by a result of Gray and Ruskuc which shows that any group G occurs as the maximal subgroup of some $\mathrm{IG}(E)$.

Given the group G, Gray and Ruskuc must make a careful choice for E and use a certain amount of well developed machinery. Our aim here is to present a short and direct proof of the same result, moreover by using a naturally occuring biordered set.

More specifically, for any free G-act $F_n(G)$ of finite rank $n \geq 3$, we have that G is a maximal subgroup of $\mathrm{IG}(E)$ where E is the biordered set of idempotents of End $F_n(G)$. Note that if G is finite then so is End $F_n(G)$. This is a joint result with Dandan Yang.