Jin Akiyama, (Tokai University)

Recent Results on Polyhedra

Studies on polyhedra have a very long history of more than 2000 years. However, there are still many interesting problems in this area.In this lecture we discuss many recent results and research problems including conjectures on the following topics:

1. Reversible Solids
2. Tile Makers and Semi Tile Makers
3. Space Filling Polyhedra with plane tiling developments
4. Polyhedron with an its cross-sections tillers
5. The minimum perimeter developments of Platonic solids
6. Atoms for Platonic solids

Noga Alon (Tel Aviv University)

Weak epsilon-nets: old problems and (some) recent progress

For a finite set X of n points in the d-dimensional Euclidean space, and a positive real ε, a subset Y of the space is a weak ε-net for X (with respect to convex bodies) if, whenever B is a convex body which is large in the sense that it contains at least εn points of X, then B contains at least one point of Y.

It is well known, though not obvious, that every X admits a weak ε-net of size at most $f(\varepsilon, d)$, which is independent of the size of X, and the problem of estimating the best possible function $f(\varepsilon, d)$, and in particular that of deciding whether or not for fixed d it is upper bounded by c_d/ε, is open despite a considerable amount of efforts. I will briefly describe some of the history of the problem, focusing on a recent joint work with Kaplan, Nivasch, Smorodinsky and Sharir, in which we show that certain natural classes of sets X (like points in convex position in the plane), admit weak ε-nets of size that exceeds $O(1/\varepsilon)$ by at most some inverse Ackermann-type function of $1/\varepsilon$.

Keith Ball (University College London)

A sharp combinatorial version of Vaaler's Theorem

Vaaler's Theorem states that every central section of a unit cube in R^n has volume at least one. We prove that if K is a subspace of R^n and $Q = [-1, 1]^n$ is the centrally symmetric cube then there is a probability measure P on $K \cap Q$ for which the operator $\int x \otimes x \, dP$ dominates the identity on K in the sense of positive definite operators. Thus the slice of the cube is "fat in all directions" rather than merely having large volume. The proof uses a sequence of linked optimisation problems to build the probability.

Alexander Barvinok (University of Michigan):

The correlation between the row and column sums of a non-negative integer matrix

Let us fix positive integers m, n, and N and let us consider the set of mxn non-negative integer matrices with the total sum N of entries as a finite probability space with the uniform counting measure. Let us consider two events in the space, one consisting of the matrices with the given m-vector of row sums R and the other consisting of the matrices with the given n-vector of column sums C. It looks intuitively rather obvious, has been conjectured, and even proved in some special cases that the two events are "almost independent". It turns out, however, that as long as R and C are both non-uniform, the events are not independent but in fact asymptotically positively correlated when m and n grow, and R and C grow "proportionately". The proof is geometric and to some extent computational, while a satisfying intuitive explanation of this phenomenon seems to be missing.

András Bezdek (University of Auburn, Rényi Institute)

On coverings where crossings are unavoidable, and other density related problems

The talk will explain the status of several problems in the area of packing and covering theory. One of the the central problems of the theory of packings and coverings is to find the most economical ways to pack (to arrange without overlap) congruent copies of a given disc within a container or to cover a container by congruent copies of a given disc. To measure optimality, the notion of *density* was introduced. Although, many far reaching theorems are known for packings, much less is know about coverings.

László Fejes Tóth conjectured in 1972 that for any convex disc C in the plane, the density $d(C)$ of the thinnest covering with congruent copies of C satisfies $d(C) \geq \frac{area(C)}{area(h_C)}$, where h_C denotes a hexagon of maximum area inscribed in C. L. Fejes Tóth showed the above inequality under the assumption that the congruent copies do not cross each other. Two convex discs C and C' are said to *cross* each other if $C \backslash C'$ and $C' \backslash C$ are disconnected. Eliminating the assumption of crossing is one of the major unsolved problems in this area.

It was felt that the condition requiring the copies to be noncrossing is merely a technical one and it was hoped that one can eliminate all crossing by rearranging some members of the covering without increasing the density. Examples of Heppes and Wegner already showed coverings of bounded containers, where such rearrangements do not exist. On the other hand, recent results of Heppes and G. Fejes Tóth showed that if the plane is to be covered, then such rearrangements exist for some special type of convex discs. Recently W. Kuperberg described a covering of the plane by special congruent pentagons and conjectured, that any covering of the plane with congruent copies of this pentagon must contain two pentagons which cross each other. In the talk we show how to approach and prove this conjecture.

Finding reasonably good upper and lower bounds for the translational packing densities for the family of short cylinders, convex cones is a typical problem in discrete geometry. Based on joint work with W. Kuperberg, we show constructions, results and several conjectures.

Károly Bezdek (University of Calgary)

On some recent progress on ball-polyhedra

The results to be discussed are centered around the following three problems:

- Finding shortest billiards in ball-polyhedra;
- Minimizing the volume of ball-polyhedra of given minimum width;
- Covering ball-polyhedra by planks.

Gábor Fejes Tóth (Rényi Institute)

Partial Covering of a Convex Domain by Translates of a Centrally Symmetric Convex Disc

Let R be a convex domain and $\mathcal{S}$ a finite family of translates of a centrally symmetric convex disc C. Let d denote the density of $\mathcal{S}$ relative to R. Further let σ be the density of the part of R covered by the members of $\mathcal{S}$ relative to R. Given d, we are looking for the maximum of σ. We give an upper bound for σ, which is asymptotically tight as the cardinality of $\mathcal{S}$ approaches infinity. Our result generalizes some old theorems of L. Fejes Tóth and Rogers.

Zoltán Füredi (University of Illinois at Urbana-Champaign & Rényi Institute)

Covers of closed curves of lenght two (joint work with J. Wetzel)

A set C in the plane is a *cover* for the family of arcs $\mathcal{A}$ if it contains a congruent copy of each arc in the family. Let $\alpha(\mathcal{A})$ be the least area of a convex C (or rather, it is an infimum). We are most interested in the problem when $\mathcal{A}$ consists of all closed curves of length two, and slightly improve the best known lower and upper bounds showing $0.386 < \alpha < 0.449$.

Peter M. Gruber (Technische Universität Wien):
Voronoi type results

Voronoi proved his famous result on lattice packings of balls which locally have maximum density by translating the ball packing problem in $\mathbb{E}^d$ into a problem on separation of convex sets in $\mathbb{E}^{d(d+1)/2}$. It turns out that a similar translation can be applied to lattice packings of general convex bodies. We study such packings with (upper) stationary, extreme and ultra extreme density and characterize packings which are upper stationary and ultra extreme. Surprisingly, there are no stationary packings. In the case of balls a second unexpected result says the each extreme packing, actually, is ultra extreme.

It is a pleasure for me to participate in the conference honouring my dear friend László (a friend since 1964, number theory conference at Ohio State).

Tom Hales (University of Pittsburg)
Some packing problems in two and three dimensions

This talk will discuss some packing problems in two and three dimensions, including problems of particular interest to László Fejes Tóth.

László Lovász (Eötvös University)
Graph representations from matrices

There are a number of interesting connections between various forms of geometric representations and combinatorial properties of graphs. Orthogonal representations, bar-and-joint representations, touching disk representations and many others have been studied and applied in very different areas. Various interesting classes of graphs like perfect graphs, outerplanar graphs, or planar graphs, can be characterized by such representations in more than one way. Such representations can be obtained from weighted forms of adjacency matrices, and vice versa. In this talk, we survey some results and unsolved problems in this area.

H. Maehara (Ryukyu University)

From line-systems to sphere-systems (joint work with N. Tokushige)

Our goal is to settle the following problem:
Problem. Is there a family of n+2 unit spheres in $\mathbb{R}^n$ that satisfies the following condition?

Each $n+1$ spheres have nonempty intersection, but the intersection of all spheres is empty.

If we allow that the radii of spheres may differ, such a family clearly exists for every $n \geq 1$. On the other hand, it is known (Maehara 1989) that, if a family of more than $n+2$ spheres of various radii in $\mathbb{R}^n$ satisfies that each $n+1$ spheres in the family have nonempty intersection, then the intersection of all spheres is also nonempty. In the unit sphere case, the answer to the above problem depends on the dimension n. There is no such family for $n = 1$, but there are many for $n = 2$. Such a family of $n+2$ unit spheres also exists for every $n \geq 4$ (Maehara and Tokushige 2006, Bezdek *et al* 2007). Then, what is the answer for $n = 3$?

Through our study, we were convinced that there is no such family of five unit spheres in $\mathbb{R}^3$, and we could prove that this assertion follows from the following rather obvious looking conjecture: "the circumsphere of a tetrahedron never encloses any of its escribed spheres" We tried to prove this conjecture, but did not succeed.

In the meanwhile, Professor Margaret M. Bayer (University of Kansas) informed us that our conjecture is a theorem proved by John Hilton Grace (1873–1958) nearly a century ago. His proof was based on an amazing idea to convert Schläfli's double six lines into "double six spheres" by Lie's linesphere transformation.

We present outlines of the proofs of Grace's theorem and the nonexistence of a family five unit spheres in $\mathbb{R}^n$ satisfying the above condition. We also present brief accounts on Schläfli's double six theorem and Lie's line-sphere transformation.

E. Makai (Rényi Institute)

Packings of strings of balls and of molecules (joint work with K. Böröczky and A. Heppes)

Let in $\mathbb{R}^3$ a *string of unit balls* be defined as the union of unit balls centred at the points $(0, 0, kd)$ (where $d \geq 2$ and k is any integer). We investigate packings of translates of this string. Otherwise said, they are packings of unit balls, consisting of translates of this string. A. Bezdek, E. M., Jr. and W. Kuperberg determined the densest packing of translates of this string, for the case $d = 2$, i.e., when the neighbouring balls in the string just touch each other. The result was of course that a densest packing is the densest lattice packing of unit balls. Today this result is already superceded by the solution of the Kepler conjecture by T. C. Hales, with S. P. Ferguson. However, for $d > 2$, the question is not covered by Hales' result. We give a solution for the first interesting interval $2 < d < 2\sqrt{2}$. (For $d = 2\sqrt{2}$ Hales' result solves the question.) The solution will be a certain lattice packing of unit balls, depending on d.

A *molecule* on the plane is the union of a unit circle and of another circle, of radius r (where $r \in (0, 1)$), touching the unit circle from outside. We investigate packings of congruent copies of molecules in the plane. They are particular packings of unit circles and circles of radius r. L. Fejes Tóth posed the question of determining the densest packing of molecules, for r close to 1. This problem seems to be quite hard, and so far has resisted the attacks to solve it. However, at the other end the problem proved to be treatable. The case $r \in (0, 2/\sqrt{3} - 1) = (0, 0.1547...)$ is trivial, the small circles have enough place in the holes of the densest lattice packing of unit circles. We show the existence of an $r_0 > 2/\sqrt{3} - 1$ (actually $r_0 = 0.17...$) such that the densest packing of congruent molecules can be determined. The solution will be a mixture of a densest lattice packing of unit circles and of another lattice of unit circles, which other lattice allows just enough place to have two circles of radius r in each of its basic rhombs, of sides 2, to form a packing.

The two problems are connected via the way of proof. The proof idea is the following. Let P be a set of points in the plane, which is an (r, R)-system, i.e., the circles of radius r about these points form a packing, and the circles of radius R about these points form a covering. Let us consider the Delone-triangulation associated to P. Then the Delone triangles can be divided to groups in such a way, that each group forms a connected set in the plane, the number of Delone triangles in a group is bounded above, and for each group, containing an obtuse Delone triangle, the average area of the triangles is at least the area of an isosceles right triangle of legs r. This implies that the average area of a Delone-triangle is at least the minimum of the area of an isosceles right triangle of legs r, and of the average area of those Delone triangles, which are not obtuse. (The construction of the groups of Delone triangles follows an idea of J. Molnár, the so called L^*-subdivision associated to P.) In many packing problems the non-obtuse Delone triangles can be easily handled, while the obtuse ones present a difficulty. This grouping of Delone triangles is one way to get rid of this difficulty. Both above problems are solved this way. Still we mention that this grouping can be done also in the spherical and hyperbolic planes, with a suitable definition of "obtuse triangle". Hopefully, this method still will have applications in other packing problems as well.

Rom Pinchasi (Technion)
On some Erdős-type problems in two and three dimensions

We present some recent solutions to old Erdos-type conjectures among which are Scott's conjecture from 1970 about the minimum number of directions determined by n points in three dimensions, two conjectures of Bezdek from around 1990 about unit circles in the plane, a conjecture of Murty from 1971 about magic configurations of points in the plane, and a conjecture of Erdos, Purdy, and Strauss (1977) about the minimum number of distinct areas of triangles determined by n points in the plane. In particular, we show how the principle of duality between points and lines in the plane and Euler's formula for planar graphs can help to solve some of these problems.

Günter Rote (Freie Universität Berlin)
Collapse (joint work with Uri Zwick)

We can simulate the collapse of an unstable tower of rigid bricks, following the laws of classical mechanics (but without friction). According to the (largely forgotten) Principle of Least Constraint (Prinzip des kleinsten Zwanges) of Gauß(1829), the accelerations of the bricks are determined by a quadratic programming problem with linear side constraints. We show that this principle lends itself readily to the discretization of the differential equations of motion, as well as to the modeling of collisions, as long as they are perfectly inelastic.

Francisco Santos (University of Cantabria)

Multi-triangulations as complexes of star polygons (joint work with Vincent Pilaud)

Maximal $(k+1)$-crossing-free graphs on a planar point set in convex position, that is, k-triangulations, have received attention in recent literature, with motivation coming from several interpretations of them. In this talk we show a new way of looking at k-triangulations, namely as complexes of star polygons. With this tool we give new, direct, proofs of the fundamental properties of k- triangulations, as well as some new results. This interpretation also opens-up new avenues of research, that we briefly explore in the last part.

Rolf Schneider (Freiburg i.Br.)

Topics in the geometry of random polytopes

The considerable progress that the theory of random polytopes has seen in recent years concentrates mostly around polytopes generated as convex hulls of random points. The equally important generation of polytopes as intersections of closed halfspaces has found less attention in geometric probability. This may be due to the belief that results on convex hulls of random points carry over to results on intersections of random halfspaces, 'just by duality'. While this is true in some cases, duality can only serve as a (nevertheless useful) heuristic in other cases. Intersections of halfspaces may also pose entirely new problems, without a dual counterpart. We give examples.

Géza Tóth (Rényi Institute)

Decomposing multiple coverings (joint work with D. Pálvölgyi)

A collection $\mathcal{C}$ of planar sets is said to be a *k-fold covering* if every point in the plane is contained in at least k members of $\mathcal{C}$. L. Fejes Tóth and J. Pach investigated the following general question. Given a k-fold covering, can it be decomposed into two coverings? The question is particularly interesting if the collection $\mathcal{C}$ consists of translates of a planar set.

A planar set T is said to be cover-decomposable if the following holds. There exists a constant $k = k(T)$ such that every k-fold covering of the plane with translates of T can be decomposed into two coverings. J. Pach conjectured that all convex sets are cover-decomposable and proved in 1986 that open, centrally symmetric, convex polygons are cover-decomposable.

We review recent developments related to this conjecture, in particular, we prove that all open convex polygons are cover-decomposable. On the other hand, we show the recent construction of D. Pálvölgyi, which proves that "most of" the concave polygons are not cover-decomposable.

Tudor Zamfirescu (Universität Dortmund)

Pushing convex and other bodies through holes