Gabriela Araujo, garaujo@math.unam.mx
A relation between Cayley Graphs and Polytopes

Given a (p, q)-graph, we will introduce a generalization of the Permutahedron using Cayley graphs that yield an abstract polytope of range q.

Ted Bisztriczky, tbisztri@math.ucalgary.ca
On convex 4-polytopes

We present a brief survey of research of the past ten years into these polytopes, with focus on problems concerning construction, classification, separation and antipodality.

Boris Bukh, bbukh@Math.Princeton.EDU
Stabbing simplices by points and affine spaces (joint work with Jiří Matoušek and Gabriel Nivasch)

Bárány showed that there is a constant $c_d > 0$ such that if S is any n-point set in R^d, then there exists a point in c_d fraction of simplices spanned by S. We present a simple construction of a point set for which there is no point contained in many simplices. The construction is optimal for $d = 2$ and gives the first non-trivial upper bounds on c_d for $d \geq 3$. We will also discuss generalizations to stabbing simplices by affine spaces.

Robert Connelly, rc46@cornell.edu

Maximizing the area of unions and intersections of disks

Suppose a finite number of disks (of any fixed dimension) are given as well as a Boolean formula in unions and intersections of those disks. Various problems concerned with maximizing the area of such Boolean expressions of those disks will be discussed.

Ludwig Danzer, danzer@mathematik.uni-dortmund.de

Some new examples of quasiperiodic tilings (apart from inflation)

Ferenc Fodor, fodorf@math.u-szeged.hu

Random approximation of convex bodies with a reasonably smooth boundary

(joint work with K.J. Böröckzy, M. Reitzner and V. Vígh)

In this talk we will review some new results about the expectation and variance of the mean width of random polytopes inscribed in convex bodies with reasonably smooth boundary. We will also give an overview of the current state of the field in this topic and point out open problems.

Jacob Fox, jacobfox@Math.Princeton.EDU

Arrangements of curves and partially ordered sets (joint work with János Pach)

A *string graph* is a graph that can be obtained as the intersection graph of curves (strings) in the plane. Despite much attention over the past half century, the structure of string graphs is poorly understood. The *incomparability graph* of a partially ordered set $(P, <)$ has vertex set P and two elements of P are adjacent if and only if they are incomparable. It is known that every incomparability graph is a string graph, and that the converse is not true. Nevertheless, we demonstrate an intimate relationship between these two types of graphs: we prove that for every collection C of curves in the plane whose intersection graph is dense, we can pick for each curve $\gamma \in C$ a subcurve γ' such that the intersection graph of the collection $\{\gamma' : \gamma \in C\}$ is a dense incomparability graph. It follows that every dense string graph has a dense subgraph which is an incomparability graph. Utilizing this connection between arrangements of curves and partially ordered sets, we make progress on several extremal problems for string graphs and crossing edge patterns in graphs drawn in the plane.

Gábor Gévay, gevay@math.u-szeged.hu

Some examples of (n_k) configurations derived from highly symmetric polytopes

An (n_k) configuration is a collection of n points and n straight lines such that each point lies on k lines and each line passes through k points. Since 1990 there has been a renaissance of research in this field. Here we present some new examples of (n_k) configurations. These are constructed from regular and uniform polytopes, hence are spatial configurations. Due to this method, they have high degree symmetry, and unusually high value of n can be attained.

Jin-ichi Itoh, j-itoh@kumamoto-u.ac.jp
Thread construction of quadratic (hyper) surfaces (joint work with Kazuyoshi Kiyohara)

Well known since old times is the thread construction of the ellipse. In 1882, O. Staude found a thread construction for the ellipsoid in 3 dimensional Euclidean space; his method was explained in the book "Intuitive Geometry" written by Hibert and Cohn-Vossen [3]. His first proof was quite complicated [4]; later he gave simpler proofs and also extended the results to the case of two-sheeted hyperboloids and hypebolic paraboloids [5], [6].

In this talk, we will give a modern proof by using the following Proposition and Theorem, which are derived from the equations of geodesics [1], [2] and explain thead constructions of all quadratic surfaces. Moreover, using similar arguments, we extend the results to quadratic hypersurfaces, similar surfaces in space forms with constant curvature and intersections of two quadratic hypersurfaces.

For $a_0 > a_1 > a_2 > 0$, put

$$C_1^{\pm} := \{(x_0, x_1, x_2) | x_1 = 0, \frac{x_0^2}{a_0 - a_1} + \frac{x_2^2}{a_2 - a_1} = 1, \pm x_0 > 0\}$$

$$C_2 := \{(x_0, x_1, x_2) | x_2 = 0, \frac{x_0^2}{a_0 - a_2} + \frac{x_1^2}{a_1 - a_2} = 1\}$$

Let L be the x_0-axis ($L := \{x_1 = x_2 = 0\}$).

For any $x \in \mathbb{R}$ there exist an ellipsoid $Q_0(x)$, a one sheeted hyperboloid $Q_1(x)$ and a two sheeted hyperboloid $Q_2(x)$, which are confocal quadratic surfaces through x with $C_1^{\pm}, C_2$ as their focal curves, such that

$$\frac{x_0^2}{a_0 - \lambda} + \frac{x_1^2}{a_1 - \lambda} + \frac{x_2^2}{a_2 - \lambda} = 1.$$

Let $q_0(x), q_1(x), q_2(x), q_0'(x), q_1'(x)$ be points $\in L$, defined by

$$L \cap Q_i(x) = \{q_i(x), q_i'(x)\} (i = 0, 1),\ L \cap Q_2(x) = \{q_2(x)\},$$

the first coordinate of $q_i(x)$ is positive and that of $q_i'(x)$ is negative ($i = 0, 1$).

Let p_1, p_2, p_2', p_1' be the points on L of first coordinates

$$\sqrt{a_0 - a_2},\ \sqrt{a_0 - a_1},\ -\sqrt{a_0 - a_1},\ -\sqrt{a_0 - a_2}.$$

(continued next page)

Proposition. *(1) On L the above points are lined in the order* $q_0, p_1, q_1, p_2, q_2, p_2', q_1', p_1', q_0'$

(2) The above correspondence $x \in \mathbb{R} \longrightarrow (q_0, q_1, q_2)$ *is locally diffeomophic.*

Theorem. *If* $x(t)$ *in* $\mathbb{R}$ *is a geodesic (i.e. a line), then the length of any segments of* $x(t)$ $(a \leq t \leq b)$ *is not less than the sum of the distances covered by the corresponding three points* $q_i(x(t))$ $(a \leq t \leq b)$ $(i = 0, 1, 2)$ *on L. The equality holds if and only if* $x(t)$ *goes through the two focal curves* $C_1^+ \cup C_1^-$ *and* C_2*.*

References

[1] J. Itoh & K. Kiyohara *On the lengths of simple closed (quasi)geodesics on convex surfaces*, Comptes Rendus Math., 343 (2006), 259-264.
[2] J. Itoh & K. Kiyohara *The cut loci on ellipsoids and certain Liouville manifolds*, Preprint.
[3] D. Hilbert & S. Cohn-Vossen *Anschauliche Geometrie*, Springer, Berlin 1932.
[4] O. Staude *Ueber Fadenconstruction des Ellipsoides*, Math. Annalen, 20 (1882), 147-184.
[5] O. Staude *Ueber neue Focaleigenschaften der Flächen 2. Grades*, Math. Annalen, 27 (1886), 253-271.
[6] O. Staude *Die algebraischen Grundlagen der Focaleigenschaften der Flächen 2. Ordnung*, Math. Annalen, 50 (1898), 398-428.

Zsolt Lángi, zlangi@math.ucalgary.ca

On the Hadwiger numbers of centrally symmetric starlike disks

The Hadwiger number $H(S)$ of a topological disk S in the plane is the maximum number of pairwise nonoverlapping translates of S that touch S. It is well known that if S is convex, then its Hadwiger number is at most 8. A. Bezdek, K. Kuperberg and W. Kuperberg conjectured that the same upper bound holds for the Hadwiger numbers of starlike disks. In 1995, A. Bezdek proved that $H(S) \leq 75$ for any starlike disk S. In this talk, I am going to sketch a proof that if S is a centrally symmetric starlike disk, then its Hadwiger number is at most 12.

Vitali Milman, milman@post.tau.ac.il

The abstract concept of duality and some examples (partly joint with Shiri Artstein-Avidan)

We discuss in the talk an unexpected observation that very minimal basic properties essentially uniquely define some classical transforms which traditionally are defined in a concrete and quite involved form. We start with a characterization of a very basic concept in Convexity: Duality and the Legendre transform. We show that the Legendre transform is, up to linear terms, the only involution on the class of convex lower semi-continious functions in R^n which reverses the (partial) order of functions. This leads to a different understanding of the concept of duality, which we call an “abstract duality concept”, and which we then apply also to many other well known settings including the result by Boroczky and Schneider. It is also true that any involutive transform (on this class) which exchanges summation with inf-convolution, is, up to linear terms, the Legendre transform. More examples and one new and unusual duality will be shown.

Luis Montejano, luis@math.unam.mx

Knesser-Lovász Hypergraphs and transversals to discrete embeddings in affine space (joint with J. Arocha, J. Bracho and J. Ramirez-Alfonsin)

Let $\mathbf{M}(\mathbf{k}, \mathbf{d}, \lambda)$ be the maximum positive integer n such that every embedding of n points in regular position in R^d has the property that the convex hull of all k-set have a transversal $(d-\lambda)$-plane and let $\mathbf{m}(\mathbf{k}, \mathbf{d}, \lambda)$ be the minimum positive integer n such that for every embedding of n points in regular position in R^d the convex hull of the k-sets does not have a transversal $(d-\lambda)$-plane.

We calculate $\mathbf{m}(\mathbf{k}, \mathbf{d}, \lambda)$ and give lower and upper bounds for $\mathbf{M}(\mathbf{k}, \mathbf{d}, \lambda)$.

Let us consider $\binom{[n]}{k}$, by the collection of $k-$ subsets of $[n] = \{1, ..., n\}$ and let us define the hypergraph $G^\lambda(n, k)$, as the hypergraph whose vertices are $\binom{[n]}{k}$, and a collection of vertices $\{S_1, ...S_\rho\}$ is a hyperedge of $G^\lambda(n, k)$ if and only if $\rho \leq \lambda$ and $S_1 \cap ... \cap S_\rho = \phi$.

So, a collection of vertices $\{S_1, ...S_\xi\}$ of the hypergraph $G^\lambda(n, k)$ is independent if and only if either $\xi \leq \lambda$ and $S_1 \cap ... \cap S_\xi \neq \phi$, or $\xi > \lambda$ and any $\lambda-$subfamily $\{S_{i_1}, ...S_{i_\lambda}\}$ of $\{S_1, ...S_\xi\}$ is such that $S_{1_1} \cap ... \cap S_{i_\lambda} \neq \phi$ (satisfies the λ-Helly property).

We give lower and upper bounds for **the** chromatic number $\chi(G^\lambda(n, k))$. We use topology, in fact, Dolnikov ideas, to related $\mathbf{M}(\mathbf{k}, \mathbf{d}, \lambda)$ and $\chi(G^\lambda(n, k))$.

Chie Nara, cnara@ktmail.tokai-u.jp

Space-filling polyhedra with reflectiveness (joint work with Jin-ich Itoh)

It is asked by Hilbert to find all polyhedra which are space-fillers. We define a property of reflectiveness for space-fillers and show that there exists only three tetrahedra, three triangular right prisms, and cuboids for space-fillers with reflectiveness among all polyhedra.

Márton Naszódi, mnaszodi@math.ualberta.ca

Covering a Convex Body by its Homothets of Different Sizes

We consider two problems. First, we find an upper bound on the quantity $f_d(K)$ defined as the least positive number such that any system $K_1, K_2, \ldots$ of positive homothets of the convex body $K \subset \mathbb{R}^d$ whose total volume is at least f permits a translative covering of K. This problem originates in a conjecture of László Fejes Tóth (1984) according to which $f_2(K) \leq 3$ for all convex planar bodies K. The previously known best upper bound that is independent of K, $f_d(K) \leq (d+1)^d - 1$, is due to Januszewski (2003). We prove that $f_d(K) \leq 6^d$.

Next, we consider the following conjecture of V. Soltan (1990): Let $K_1, K_2, \ldots$ be a system of homothets of the convex body $K \subset \mathbb{R}^d$ with the property that the coefficients of homothety are strictly between 0 and 1. If the system covers K then the sum of the coefficients of homothety is at least d. We confirm a weaker, asymptotic version of this conjecture.

Deborah Oliveros, dolivero@matem.unam.mx

Some realizations and symmetries of the Graphicahedron

The Graphicahedron is an abstract polytope that generalize in a sense the Permutahedron. We will discuss some realization and symmetries of such polytope.

Hellmuth Stachel, stachel@dmg.tuwien.ac.at
Global rigidity of a simplex in Euclidean 4-space with prescribed areas of 2-facets

Due to the Cayley-Menger-formula the volume of any simplex can be expressed in terms of its edge lengths. In dimension four the edges are dual to the 2-faces. Hence there might be a 'dual' formula expressing the simplex volume by the areas of its ten 2-faces. The aim of this paper is to discuss the uniqueness of a simplex in Euclidean 4-space with given 2-areas.

Ricardo Strausz, ricardo.strausz@gmail.com
Open problems on separoids

A separoid is a symmetric relation defined in disjoint pairs of subsets which is closed as a filter in the canonical order induced by the inclusion. Different objects, like (simple) graphs, polytopes and oriented matroids, find a common generalisation in this framework. All separoids can be represented by families of convex sets, in some Euclidian space, and their separations by hyperplanes ?hence the name of the structure. In the present lecture we will revise some results and open problems of this theory.

Konrad Swanepoel, konrad.swanepoel@gmail.com
Tightest packing and loosest covering in infinite dimensional spaces

The equivalent notions of tightest packing and loosest covering were introduced by Rogers, Ryskov and L. Fejes Tóth. Important results were obtained by Butler, Böröczky, Linhardt, Zong, Henk and others. Adapting an old construction of Victor Klee we consider tight packings of the unit ball of the infinite dimensional ℓ_p spaces.

Liping Yuan, lpyuan88@yahoo.com
Acute triangulations of surfaces

An *acute* triangulation is a triangulation whose triangles have all their angles less than $\frac{\pi}{2}$. In this talk acute triangulations of several surfaces are discussed.

Asia Ivic Weiss, weiss@yorku.ca
Map operations and k-orbit maps

A map with exactly k flag-orbits under the action of its automorphism group is called a k-orbit map. We determine the classes of k-orbit maps for $k \leq 4$ and describe the tools to determine the classes in general. We also investigate medials and truncations of such maps.

Oloff de Wet, oloffdewet@gmail.com
The Euclidean Steiner Ratio

An Euclidean Steiner tree is a tree, embedded in the Euclidean plane, which interconnects a given set of vertices. Finding such a tree of minimal length, called a Steiner minimal tree, is not a trivial task. (The problem is generally NP-complete.) We can compare the lengths of Steiner minimal trees with those of minimum spanning trees (which many algorithms can find in polynomial time) by using the Steiner ratio.

In 1992 Du and Hwang published a paper confirming the correctness of a well known 1968 conjecture of Gilbert and Pollak suggesting that the Euclidean Steiner ratio for n points in the plane is $2/\sqrt{3}$. We take a critical look at this paper and provide counterexamples to expose a fundamental mistake. We conclude by applying their strategy to prove the validity of the Gilbert-Pollak conjecture for 7 points.