5TH WORKSHOP ON REAL AND COMPLEX HADAMARD MATRICES

Budapest, Hungary

July 10–14, 2017

Book of abstracts

http://www.renyi.hu/conferences/hadamard2017

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Welcome to the 5th workshop on real and complex Hadamard matrices!

The Rényi Institute of Mathematics announces an international workshop on Real and Complex Hadamard Matrices and Applications, Budapest, Hungary, 10-14 July, 2017. This workshop is a continuation of the series of “Hadamard workshops” held earlier in Seville (2007), Galway (2009), Melbourne (2011), and Lethbridge (2014).

The workshop focuses on the theory and applications of real and complex Hadamard matrices.

Information for participants:

Wifi is available at our Institute. The password will be distributed in your conference package. If you require specifically, we can also set up a temporary account for you, so that you can use the computer terminals of the Institute.

The exchange rate between EUR and HUF is currently around 308. Therefore, if you are ever offered a rate below 290 you can be sure that you are being cheated. There are good exchange offices nearby the Institute throughout downtown Budapest; moreover most ATMs accept most major debit/credit cards to withdraw cash. You are advised that tipping is unfortunately anticipated in Hungary for services such as taxi rides, barber shops, and also in bars and non-self-service restaurants. The 10% tip is usually paid in addition to the amount shown on your bill.

There are no talks on Wednesday afternoon, and we encourage you to enjoy some of the tourist attractions of Budapest.

Organizers:

- Máté Matolcsi (Chair)
- Ferenc Szöllősi

Future workshops

We hope that the series will continue in the next 2-3 years, hopefully in a destination where it has not been held insofar. Participants willing to take the burden of organizing a prospective workshop in the foreseeable future are encouraged to step forward during the workshop.[1]

[1] Note added after the event: The 6th workshop is scheduled to take place in Kraków, Poland in 2020. We thank Karol Życzkowski for stepping up and willing to carry the torch.

List of participants:

- Victor Álvarez, Universidad de Sevilla, Spain
- José Armario, Universidad de Sevilla, Spain
- Ivan Bailera, Universitat Autònoma de Barcelona
- Santiago A. Barrera, Monash University, Australia
- Ádám Besenyei, ELTE University, Hungary
- Alexey Bondal, Steklov Math. Institute; and Higher School of Economics, Russia
- Wojciech Bruzda, Jagiellonian University, Poland
- Barbara Ciesielska, Jagiellonian University, Poland
- Charles Colbourn, Arizona State University, United States
- Robert Craigen, University of Manitoba, Canada
- Dean Crnković, University of Rijeka, Croatia
- Ronan Egan, University of Rijeka, Croatia
- Raul M. Falcón Ganfornina, Universidad de Sevilla, Spain
- Máté Farkas, University of Gdańsk, Poland
- Dane Flannery, National University of Ireland, Ireland
- Maria D. Frau, Universidad de Sevilla, Spain
- Péter Frenkel, ELTE University, Hungary
- Dardo Goyeneche, Jagiellonian University, Poland
- Markus Grassl, Max Planck Institute for the Science of Light, Germany
- Felix Gudiel, Universidad de Sevilla, Spain
- Belen Güemes, Universidad de Sevilla, Spain
- Kathy J. Horadam, RMIT University, Australia
- Takuya Ikuta, Kobe Gakuin University, Japan
- Bengt R. Karlsson, Uppsala University, Sweden
- Hadi Kharaghani, University of Lethbridge, Canada
- Gergely Kiss, University of Luxembourg, Luxembourg
- Ilias Kotsireas, Wilfrid Laurier University, Canada
- Pekka Lampio, Aalto University, Finland
- Zoltán Léka, Royal Holloway, University of London, UK
- Máté Matolcsi, Budapest University of Technology and Economics, Hungary
- Milán Mosonyi, Budapest University of Technology and Economics, Hungary
- Akihiro Munemasa, Tohoku University, Japan
- Padraig Ó Cathain, Worcester Polytechnic Institute, United States
- William P. Orrick, Indiana University, United States

- József Pitrik, Budapest University of Technology and Economics, Hungary
- Bernhard Schmidt, Nanyang Technological University, Singapore
- Jennifer Seberry, University of Wollongong, Australia
- Anna Szczpanek, Jagiellonian University, Poland
- Ferenc Szöllősi, Aalto University, Finland
- Anna Szymusiak, Jagiellonian University, Poland
- Tamás Titkos, Rényi Institute of Mathematics, Hungary
- Mercè Villanueva, Universitat Autònoma de Barcelona, Spain
- Dániel Virosztek, Budapest University of Technology and Economics, Hungary
- Ian Wanless, Monash University, Australia
- Stefan Weigert, The University of York, United Kingdom
- Mihály Weiner, Budapest University of Technology and Economics, Hungary
- Oğuz Yayla, Hacettepe University, Turkey
- Ilya Zhdanovskiy, Moscow Inst. of Physics; and Higher School of Economics, Russia
- Karol Życzkowski, Jagiellonian University; and Center for Theor. Physics, Poland

Overview of the program

Times shown here are indicative (see next pages). **Invited** talks are 40 min. long.

	Monday	Tuesday	Wednesday	Thursday	Friday
08:45-09:15	*Registration*	*Registration*			
09:15-09:20	*Opening remarks*				
09:20-10:00	**Munemasa**	**Życzkowski**	**Bondal**	**Craigen**	**Wanless**
10:05-10:30	*Break*	*Break*	Zhdanovskiy	*Break*	*Break*
10:30-11:10	**Schmidt**	**Grassl**	*Break*	**Crnković**	**Flannery**
11:15-11:55	**Ó Cathain**	**Weigert**	**Colbourn**	**Seberry**	**Armario**
12:00-14:00	*Lunch*	*Lunch*	**Orrick**	*Lunch*	*Lunch*
14:00-14:40	**Kotsireas**	**Karlsson**	*Group photo*	**Horadam**	Falcón
14:45-15:05	Barrera	Matolcsi	Free afternoon	Bailera	**Álvarez**
15:10-15:30	Yayla	Goyeneche		Villanueva	Gudiel
15:35-16:00	*Break*	*Break*		*Break*	
16:00-16:20	Egan	Farkas		Lampio	
16:25-16:45	Ikuta	Szymusiak		Szöllősi	
16:50-17:10		Szczepanek		*Banquet*	

Schedule for Monday

Monday
08:45–09:15 *Registration*
09:15–09:20 *Opening remarks*
Chair: Matolcsi
09:20–10:00 Munemasa **A matrix approach to Yang multiplication**
10:05–10:30 *Coffee break*
Chair: Matolcsi
10:30–11:10 Schmidt **Weil Numbers, Bent Functions, and Circulant Hadamard Matrices**
11:15–11:55 Ó Cathain **Morphisms of Butson matrices**
12:00–14:00 *Lunch break*
Chair: Munemasa
14:00–14:40 Kotsireas **Low Autocorrelation Binary Sequences (LABS)**
14:45–15:05 Barrera Perfect Sequences over the Quaternions and Relative Difference Sets in $\mathbb{Z}_n \times Q_8$
15:10–15:30 Yayla On Near Butson-Hadamard matrices
15:35–16:00 *Coffee break*
Chair: Kotsireas
16:00–16:20 Egan Phased unitary Golay pairs, Butson Hadamard matrices and a conjecture of Ito's
16:25–16:45 Ikuta Butson-type complex Hadamard matrices and association schemes

Schedule for Tuesday

Tuesday
09:00–09:20 *Registration*
Chair: Matolcsi
09:20–10:00 Życzkowski **Complex Hadamard matrices with a special structure**
10:05–10:30 *Coffee break*
Chair: Matolcsi
10:30–11:10 Grassl **Small Unextendible Sets of Mutually Unbiased Hadamard Matrices**
11:15–11:55 Weigert **Mutually unbiased product bases**
12:00–14:00 *Lunch break*
Chair: Szöllősi
14:00–14:40 Karlsson **Complex Hadamard Matrices for** $N = 6$
14:45–15:05 Matolcsi Non-existence of certain MUB-systems in dimension 6
15:10–15:30 Goyeneche Any SIC-POVM defines a complex Hadamard matrix in every dimension
15:35–16:00 *Coffee break*
Chair: Życzkowski
16:00–16:20 Farkas n-fold unbiased bases: an extension of the MUB condition
16:25–16:45 Szymusiak The informational power of the Hoggar SIC-POVM
16:50–17:10 Szczepanek Quantum dynamical entropy, chaotic unitaries and complex Hadamard matrices

Schedule for Wednesday

Wednesday
Chair: Matolcsi
09:20–10:00 Bondal **Geometric and categorical aspects of Hadamard matrices**
10:05–10:25 Zhdanovskiy Commutators of projectors, mutually unbiased bases and projective geometry
10:30–10:55 *Coffee break*
Chair: Szöllősi
10:55–11:35 Colbourn **Asymptotic and Constructive Bounds for Covering Arrays**
11:40–12:20 Orrick **On the generation of skew Hadamard matrices**
12:25–12:30 *Group photo*
12:30– *Free afternoon*

Excursion

On Wednesday afternoon a sightseeing walking tour in Budapest and/or a visit to a thermal bath will be offered for the participants (please bring your swimming suits). More details will be available at the workshop.

Banquet

The conference dinner will take place on July 13 (Thursday), 18:00-20:30. More details will be available at the workshop.

Schedule for Thursday

Thursday
Chair: Matolcsi
09:20–10:00 Craigen **Synthetic Orthogonality Theory – To go where math has not gone before**
10:05–10:30 *Coffee break*
Chair: Craigen
10:30–11:10 Crnković **A construction of regular Hadamard matrices and related codes**
11:15–11:55 Seberry **Hadamard Matrices and Clifford-Gastineau-Hills Algebras**
12:00–14:00 *Lunch break*
Chair: Crnković
14:00–14:40 Horadam **Codes from groups and groups from codes**
14:45–15:05 Bailera Hadamard full propelinear codes of type CQ
15:10–15:30 Villanueva Partial permutation decoding for $\mathbb{Z}_{2^k}$-linear Hadamard codes
15:35–16:00 *Coffee break*
Chair: Ó Cathain
16:00–16:20 Lampio Classification of Butson-type Hadamard matrices using an orderly algorithm
16:25–16:45 Szöllősi Open probelms on Butson matrices
18:00–20:30 *Banquet*

Schedule for Friday

Friday
Chair: Matolcsi
09:20–10:00 Wanless **Congruences for the Number of Transversals of Latin squares**
10:05–10:30 *Coffee break*
Chair: Horadam
10:30–11:10 Flannery **Quasi-orthogonal cocyclic matrices**
11:15–11:55 Armario **Generalized binary sequences from quasi-orthogonal cocycles**
12:00–14:00 *Lunch break*
Chair: Flannery
14:00–14:20 Falcón Cocyclic Hadamard matrices over Latin rectangles
14:25–15:05 Álvarez **(Pseudo)-cocyclic (structured) Hadamard matrices over (quasi)groups**
15:10–15:30 Gudiel Gröbner bases and cocyclic Hadamard matrices

(Pseudo)-cocyclic (structured) Hadamard matrices over (quasi)groups

Victor Álvarez
Universidad de Sevilla, Spain

FRI
14:25

Abstract

Progressing on a survey of the state of the art of cocyclic constructions for Hadamard matrices, we will prove that the family of Goethals-Seidel Hadamard matrices is (pseudo)-cocyclic over a certain family of quasigroups. Therefore, what we call (pseudo)-cocyclic (structured) Hadamard matrices over (quasi)groups, precisely those matrices satisfying some kind of cocycle Hadamard test known so far, include most of the well-known and prolific constructions of Hadamard matrices. We will discuss also different approaches in order to look for large matrices of this type.

This is joint work with J.A. Armario, R.M. Falcón, M.D. Frau, M.B. Güemes, F. Gudiel and I. Kotsireas.

Generalized binary sequences from quasi-orthogonal cocycles

José A. Armario
Universidad de Sevilla, Spain

FRI
11:15

Abstract

Generalized Perfect Binary Arrays (GPBAs) were introduced by Jedwab as a useful tool in the construction of perfect binary arrays. It is well-known that a nontrivial GPBA exists only if its energy is either 2 or a multiple of 4. In this talk we introduce the notion of Generalized Optimal Binary Arrays (GOBAs) for binary arrays with energy $4t+2$. They are the analogous to the GPBAs. We establish a characterization of GOBAs in terms of cocycles. This determines a procedure for constructing such arrays. Finally, we provide some negaperiodic Golay pairs from generalized optimal binary sequences for small lengths.

This is joint work with Dane Flannery, National University of Ireland.

Hadamard full propelinear codes of type CQ

THU
14:45

Ivan Bailera
Universitat Autònoma de Barcelona, Spain

Abstract

A code C of type CQ is defined by a group isomorphic to $C_t \times Q$, where C_t is the cyclic group of order t and Q is the quaternion group. We study Hadamard full propelinear codes which are equivalent to Hadamard groups. The codes of type CQ have a group of permutations isomorphic to $C_t \times C_2^2$. The cocycles over the groups $C_t \times C_2^2$, for t odd, were studied recently by Baliga and Horadam. The solution set includes all Williamson Hadamard matrices, so this set of groups is potentially a uniform source for generation of Hadamard matrices. In our work, concepts such as rank and dimension of the kernel are studied, and bounds are established.

This is joint work with J. Borges and J. Rifà.

Perfect Sequences over the Quaternions and $(4n, 2, 4n, 2n)$-Relative Difference Sets in $\mathbb{Z}_n \times Q_8$

MON
14:45

Santiago Barrera Acevedo
Monash University, Australia

Abstract

The periodic autocorrelation of a sequence is a measure for how much the sequence differs from its cyclic shifts. If the autocorrelation values for all nontrivial cyclic shifts are 0, then the sequence is perfect. It is well known that sequences with good autocorrelation properties, such as being perfect, have important applications in information technology. However, it is very difficult to construct perfect sequences over 2-nd, 4-th, and in general over nth roots of unity. In fact, it is conjectured that perfect sequences over nth roots of unity do not exist for lengths greater that n^2. Due to the importance of perfect sequences and the difficulty to construct them over nth roots of unity, there has been some focus on other classes of sequences with good autocorrelation. One of these classes is the family of perfect sequences over the quaternions. In this talk I will introduce perfect sequences over the quaternion groups Q_8 and Q_{24}. These sequences exhibit interesting symmetry patterns, which I aim to explain via a connection with relative difference sets and Williamson Hadamard matrices.

This is joint work with Heiko Dietrich.

Geometric and categorical aspects of Hadamard matrices

Alexey Bondal
Steklov Math. Institute; and Higher School of Economics, Russia

WED
09:20

Abstract

I will discuss a point of view on complex Hadamard matrices and their generalizations based on Algebraic Geometry, Symplectic Geometry and discrete Harmonic analysis. I will also outline the role of Representation theory in the subject and relevant categorical constructions.

Asymptotic and Constructive Bounds for Covering Arrays

Charles J. Colbourn
Arizona State University, United States

WED
10:55

Abstract

Covering arrays are used to test the correctness of complex engineered systems with k components each having v options, when collections of at most t component options can cause failures. Asymptotic existence results bounding the sizes of covering arrays as a function of the number of components have been of much interest. For decades, the only real improvement on the simple probabilistic argument used the Lovász Local Lemma, but these probabilistic arguments had limited impact on the explicit construction of covering arrays for practical use.

Recently, many improvements on the asymptotic bounds have been obtained by Godbole, Francetic and Stevens, and Sarkar and the presenter. The methods to obtain these employ varying the sampling strategy, reducing the sample space, and oversampling with postprocessing. We outline these methods and discuss their asymptotic and constructive consequences. Despite these substantial improvements, the binary case (when $v = 2$) has seen little change. Connections with Hadamard matrices suggest a way forward, which we briefly discuss.

Synthetic Orthogonality Theory – To go where math has not gone before

THU
09:20

Robert Craigen
University of Manitoba, Canada

Abstract

As we all recall, de Launey and Flannery have made an exciting expedition to a new world of abstract combinatorial orthogonality as the first leg of a trip to conquer the relatively new territory of Algebraic Design Theory. Having travelled with them to the first destination, once the ship was landed I went rogue to explore Planet Orthogonality without a map or a predetermined goal, charting the terrain as it is encountered, without presupposing the end of establishing a bridgehead for algebra. I find many pretty stones. This talk is a recounting of my adventure with some curious artifacts on display.

A construction of regular Hadamard matrices and related codes

THU
10:30

Dean Crnković
University of Rijeka, Croatia

Abstract

A Hadamard matrix is called regular if the row and column sums are constant. The existence of a regular Hadamard matrix is well known to be equivalent to the existence of a symmetric $(4m^2, 2m^2 - m, m^2 - m)$ design, also known as a Menon design. It is conjectured that a regular Hadamard matrix of order $4m^2$ exists for every positive integer m. In this talk we give a method of constructing regular Hadamard matrices using conference graphs and Hadamard designs with skew incidence matrices. We will also discuss codes related to these Hadamard matrices, and block designs constructed from the codes.

Phased unitary Golay pairs, Butson Hadamard matrices and a conjecture of Ito's

Ronan Egan MON
University of Rijeka, Croatia 16:00

Abstract

Pairs of complementary binary or quaternary sequences of length v such as Golay pairs, complex Golay pairs and periodic Golay pairs may be used to construct Hadamard matrices and complex Hadamard matrices of order $2v$. In this talk I will define unitary Golay pairs and phased unitary Golay pairs of length v with entries in the k^{th} roots of unity for any $k \geq 2$. This generalization will lead to a construction of Buston Hadamard matrices of order $2v$ over the k^{th} roots of unity for any even k. Further, I will demonstrate how this construction strengthens a conjecture of Ito's, and consequently with this method we can construct a complex Hadamard matrix of order $2v$ for all $v \leq 46$.

Cocyclic Hadamard matrices over Latin rectangles

Raul Manuel Falcón Ganfornina FRI
Universidad de Sevilla, Spain 14:00

Abstract

In the literature, the theory of cocyclic Hadamard matrices has always been developed over finite groups. In this talk, we introduce the natural generalization of this theory to be developed over Latin rectangles. In this regard, once we introduce the concept of binary cocycle over a given Latin rectangle, we expose examples of Hadamard matrices that are not cocyclic over finite groups but they do over Latin rectangles. Since it is also shown that not every Hadamard matrix is cocyclic over a Latin rectangle, we focus on answering both problems of existence of Hadamard matrices that are cocyclic over a given Latin rectangle and also its reciprocal, that is, the existence of Latin rectangles over which a given Hadamard matrix is cocyclic. We prove in particular that every Latin square over which a Hadamard matrix is cocyclic must be the multiplication table of a loop (not necessarily associative).

This is joint work with V. Álvarez, M.D. Frau, M.B. Güemes, F. Gudiel.

n-fold unbiased bases: an extension of the MUB condition

TUE
16:00

Máté Farkas
University of Gdańsk, Poland.

Abstract

Mutually unbiased bases (MUBs) are a notion widely used in the field of quantum information theory. They first appeared in the context of quantum state determination, and then later found applications in entropic uncertainty relations, quantum key distribution, the mean king's problem, and recently quantum random access codes (QRACs). An equivalent formulation of the MUB condition is in terms of complex Hadamard matrices, thus their classification is closely related to the classification of complex Hadamard matrices.

Extending the technique used to show MUB optimality in a special case of QRACs, one can show that the optimal measurement bases in a general QRAC have to satisfy a new condition, which is a generalization of the MUB condition on n bases. This I call the nUB conditions, and the corresponding bases *n-fold unbiased*. This is an extension of the MUB condition in the sense that the $n = 2$ case corresponds to MUBs.

A useful result on nUBs is that if a set of n bases form an nUB, then any subset of $n-1$ bases forms an $(n-1)$UB. This eventually leads to the observation that every pair of bases in an nUB should also form MUBs. Therefore, when looking for nUBs, one can restrict themselves to MUBs, that are extensively studied, especially in low dimensions. Exploiting this, one can see that there exist a set of 3UBs in dimension 2, but no nUBs in dimensions $3-5$, for any $n > 2$. While this could point to a conjecture about the non-existence of these bases in general, in high dimensions some probabilistic arguments support the possibility of their existence.

Irrespective of existence, nUBs could have some applications and raise some fundamental questions about quantum mechanics. Their applications in upper bounding QRAC success probabilities is clear, and they could also be used to put bounds on other protocols usually related to MUBs. Regarding foundations, they pose questions about the geometry of quantum states, and the presence of genuine higher-order interference in physical theories.

Quasi-orthogonal cocyclic matrices

FRI
10:30

Dane Flannery
National University of Ireland, Ireland

Abstract

We introduce the notion of *quasi-orthogonal cocycle*. This is partially motivated by the maximal determinant problem for $\{\pm 1\}$-matrices of size congruent to 2 modulo 4. Quasi-orthogonal cocycles are analogous to the *orthogonal cocycles* of algebraic design theory. In line with this analogy, equivalences with new and known combinatorial objects, such as quasi-Hadamard groups, relative quasi-difference sets, and certain partially balanced incomplete block designs, are established.

This is joint work with José Andrés Armario, University of Seville.

Any SIC-POVM defines a complex Hadamard matrix in every dimension

Dardo Goyeneche TUE
Jagiellonian University, Poland 15:10

Abstract

A Symmetric Informationally Complete Positive Operator Valued Measure (SIC-POVM) is a set of d^2 normalized and equiangular complex vectors in dimension d. We demonstrate that any existing SIC-POVM in dimension d defines an hermitian complex Hadamard matrix of size d^2. In particular, the 1-parametric family of SIC-POVM existing in dimension 3 defines the maximal family of hermitian complex Hadamard matrices of size 9, which belongs to the Fourier family $F_9^{(4)}(a,b,c,d)$. Also, a special kind of SIC-POVM in dimension 8, so called the *Hoggar lines*, defines a real Hadamard matrix of size 64. Furthermore, we show that the problem to determine whether a given complex Hadamard matrix defines a SIC-POVM is a combinatorial problem.

This is joint work with M. Appleby, I. Bengtsson and S. Flammia.

Small Unextendible Sets of Mutually Unbiased Hadamard Matrices

Markus Grassl TUE
Max Planck Institute for the Science of Light, Germany 10:30

Abstract

In quantum information, Mutually Unbiased Bases (MUBs) correspond to sets of pairwise complementary observables. Fixing one of the bases, the other bases correspond to complex Hadamard matrices. Moreover, a certain product of any two of those complex Hadamard matrices yields again a Hadamard matrix. The maximal number of such bases in a system of dimension d is $d+1$, and construction of maximal sets achieving this bound are known only if the dimension is a prime power. For other dimensions, we have a lower bound of three bases, and for infinitely many dimensions, we do not know how to improve this lower bound. For specific constructions, we can show that they do not achieve the upper bound. On the other hand, even in prime power dimensions where the maximal number of MUBs can be constructed, there are unextendible sets of smaller size. The smallest possible example, a pair of MUBs based on Bachelor Hadamard Matrices, has been found in dimension 6. We present results about small sets in other dimensions, including prime powers and even primes, supporting the conjecture that there are unextendible triples of MUBs in every dimension, corresponding to unextendible pairs of Mutually Unbiased Hadamard matrices.

Gröbner bases and cocyclic Hadamard matrices

FRI
15:10

Felix Gudiel
Universidad de Sevilla, Spain

Abstract

Hadamard ideals were introduced in 2006 as a set of nonlinear polynomial equations whose zeros are uniquely related to Hadamard matrices with one or two circulant cores of a given order. Based on this idea, the cocyclic Hadamard test enables us to describe a polynomial ideal that characterizes the set of cocyclic Hadamard matrices over a fixed finite group G of order $4t$. Nevertheless, the complexity of the computation of the reduced Gröbner basis of this ideal is excessive even for very small orders. In order to improve the efficiency of this polynomial method, we take advantage of some recent results on the inner structure of a cocyclic matrix to describe an alternative polynomial ideal that also characterizes the mentioned set of cocyclic Hadamard matrices over G.

This is joint work with V. Álvarez, J.A. Armario, R.M. Falcón, M.D. Frau.

Codes from groups and groups from codes

THU
14:00

Kathy J. Horadam
RMIT University, Australia

Abstract

For 20 years I have been interested in binary codes which can be derived from cocycles on groups. I will briefly describe the *coboundary codes* defined from any function $f : \mathbb{Z}_n^2 \to \mathbb{Z}_n^2$ by $\mathcal{D}_f = \{f(x) + f(y) + f(x+y),\ x, y \in \mathbb{Z}_n^2\}$ and give some results (due to Mercè Villanueva and myself) and open questions.

Just recently, I have come across a delightful 1989 construction due to Rifà, Basart and Huguet of a group structure on *propelinear* codes, that is, subsets C of $\mathbb{Z}_n^2$ containing the all-zero codeword such that for each codeword $x \in C$ there exists a coordinate permutation $\pi \in S_n$, with $\pi_0 = \mathrm{Id}$, satisfying the conditions: (i) For all $y \in C, x + \pi_x(y) \in C$, (ii) For all $x, y \in C, \pi_x\pi_y = \pi_z$, where $z = x + \pi_x(y)$. This group is linked by work of Rifà and Suárez to cocyclic Hadamard matrices (my first love) in a most intriguing way.

Butson-type complex Hadamard matrices and association schemes on the Galois rings of characteristic 4

Takuya Ikuta
Kobe Gakuin University, Japan

MON
16:25

Abstract

A Butson-type complex Hadamard matrix is a square matrix W whose entries are roots of unity and satisfies $W\overline{W}^T = nI$, where I is the identity matrix of order n. In this talk, we give a nonsymmetric association scheme $\mathfrak{X}$ of class 6 on the Galois ring of characteristic 4, and classify hermitian complex Hadamard matrices belonging to the Bose–Mesner algebra of $\mathfrak{X}$. We show that such a matrix is necessarily a Butson-type complex Hadamard matrix whose entries are 4-th roots of unity.

This is joint work with Akihiro Munemasa.

Complex Hadamard Matrices for $N = 6$

Bengt R. Karlsson
Uppsala University, Sweden

TUE
14:00

Abstract

It is shown how to construct closed form expressions for the matrix elements of two 4-parameter orbits of complex Hadamard matrices in six dimensions.

Low Autocorrelation Binary Sequences (LABS)

MON
14:00

Ilias Kotsireas
Wilfrid Laurier University, Canada

Abstract

We will describe the LABS problem, a challenging optimization problem that arises in mathematics, communications engineering and statistical physics. We will discuss the state-of-the-art algorithmic techniques to solve this problem as well as some complexity estimates derived from experimental work by various authors. The algorithmic techniques used in the LABS problem include branch and bound methods, group theory and high-performance (parallel) computing. We will also mention the open problems in the realm of LABS, as well as some recent new ideas.

Classification of Butson-type Hadamard matrices using an orderly algorithm

THU
16:00

Pekka H.J. Lampio
Aalto University, Finland

Abstract

Butson-type Hadamard matrices are a generalization of Hadamard matrices. They are $n \times n$ complex matrices where the entries are qth roots of unity (complex numbers that are solutions to the equation $x^q = 1$), and the rows and columns are pairwise orthogonal, that is,

$$\mathbf{H}\overline{\mathbf{H}}^T = n\mathbf{I}.$$

In this computer-aided work small Butson-type Hadamard matrices are classified up to equivalence. We show how the method of orderly generation can be used efficiently for the classification of Butson-type Hadamard matrices. The main results include the enumeration of BH($n{=}21, q{=}3$), BH($n{=}16, q{=}4$) and BH($n{=}13, q{=}6$) matrices. Other new classification results are presented for BH(n, q) matrices with $n \leq 21$ and $q \leq 17$.

This is joint work with Patric R.J. Östergård and Ferenc Szöllősi.

Non-existence of certain MUB-systems in dimension 6

Máté Matolcsi
Alfréd Rényi Institute of Mathematics, Hungary

TUE
14:45

Abstract

It is widely believed that a complete system of mutually unbiased bases (or, equivalently, mutually unbiased complex Hadamard matrices) does not exist in dimension 6. We prove a partial result in this direction: any complex Hadamard matrix from the Fourier family $F_6(a,b)$ cannot be part of a complete system of MUH's.

This is based on joint work with M. Weiner.

A matrix approach to Yang multiplication

Akihiro Munemasa
Tohoku University, Japan

MON
09:20

Abstract

A quadruple of (± 1)-sequences $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d})$ of length m, m, n, n, respectively, is called *base sequences* if $N_{\boldsymbol{a}}(j)+N_{\boldsymbol{b}}(j)+N_{\boldsymbol{c}}(j)+N_{\boldsymbol{d}}(j) = 0$ for all positive integers j, where $N_{\boldsymbol{s}}(j) = \sum_{i=0}^{l-j-1} s_i s_{i+j}$ if $0 \leq j < l$, 0 otherwise, for $\boldsymbol{s} = (s_0, \ldots, s_{l-1}) \in \{\pm 1\}^l$. We denote by $BS(m,n)$ the set of base sequences of length m, m, n, n. Yang proved that, if $BS(m+1,m) \neq \emptyset$ and $BS(n+1,n) \neq \emptyset$, then $BS(m',m') \neq \emptyset$ with $m' = (2m+1)(2n+1)$.

Yang's theorem could be used to settle the restricted version of Hadamard conjecture which claims Hadamard matrices of order $8m$ exist for every odd integer m. Indeed, a class of sequences called T-sequences with length $2m'$ can be obtained from $BS(m',m')$ and Hadamard matrices of order $8m'$ can be produced from T-sequences with length $2m'$ by using Goethals–Seidel arrays.

In this talk, we use two-variable Laurent polynomials attached to matrices to encode properties of compositions of sequences. The Lagrange identity in the ring of Laurent polynomials is then used to give a short and transparent proof of Yang's theorem. We also present a generalization of another result of Yang about the construction of paired ternary sequences from base sequences. Here we use a modified version of the Lagrange identity in the ring of Laurent polynomials, together with the matrix approach developed in the first part.

This is based on joint work with Pritta Etriana Putri.

Morphisms of Butson matrices

MON
11:15

Padraig Ó Cathain
Worcester Polytechnic Institute, United States

Abstract

We introduce conditions on a pair of Butson matrices which are sufficient for the existence of a new Kronecker-type product in which the matrix entries lie in a smaller field than those of the Kronecker product. Our results generalize Turyn's construction of real matrices from matrices over fourth roots of unity, and the Compton-Craigen-de Launey construction of real Hadamard matrices from 'unreal' matrices over 6th roots of unity. We also construct new examples of maps between classes of Butson matrices over varying roots of unity, and give some applications of our method to the construction of real Hadamard matrices.

On the generation of skew Hadamard matrices

WED
11:40

William P. Orrick
Indiana University, United States

Abstract

Skew Hadamard matrices are essential ingredients in many Hadamard-matrix construction methods and are important combinatorial objects in their own right. I will survey known results on construction of skew Hadamard matrices and on the classification of skew Hadamard matrices of small size. Given a general Hadamard matrix, one can ask whether it is Hadamard equivalent to a skew Hadamard matrix. I will explain how one can answer this question and then use the method to determine what fraction of equivalence classes in existing databases contain skew Hadamard matrices. It is known that large numbers of Hadamard matrix equivalence classes can be generated by switching operations. The same turns out to be true of skew Hadamard matrices; I will describe some techniques that have proved effective in this regard.

Weil Numbers, Bent Functions, and Circulant Hadamard Matrices

Bernhard Schmidt
Nanyang Technological University, Singapore

MON
10:30

Abstract

Let ζ_m be primitive complex root of unity and let n be a positive integer. An element X of $\mathbb{Z}[\zeta_m]$ is called an ***n*-Weil number** if $|X|^2 = n$. We call X **trivial** if $X = \eta\sqrt{n}$ for some root of unity η. I will present the following results.

Let p be an odd prime and let a, b be a positive integers.

(1) If n is a nonsquare and an n-Weil number in $\mathbb{Z}[\zeta_{p^a}]$ exists, then $n^2+n+1 \geq p$.

(2) If $n = q^{2b}$ where $q \neq p$ is a prime and a nontrivial n-Weil number in $\mathbb{Z}[\zeta_{p^a}]$ exists, then

$$q^b \geq \frac{1 + \mathrm{ord}_p(q)}{2}.$$

I will explain how these results can be used to establish nonexistence results for generalized bent functions and circulant Hadamard matrices.

Hadamard Matrices and Clifford-Gastineau-Hills Algebras

Jennifer Seberry
University of Wollongong, Australia

THU
11:15

Abstract

Research into the construction of Hadamard matrices and orthogonal designs has led to deeper algebraic and combinatorial concepts. This paper surveys the place of amicability, repeat designs and the Clifford and Clifford-Gastineau-Hills algebras in laying the foundations for a *Theory of Orthogonal Designs*.

Quantum dynamical entropy, chaotic unitaries and complex Hadamard matrices

TUE
16:50

Anna Szczepanek
Jagiellonian University, Poland

Abstract

We consider successive measurements performed on a d-dimensional quantum mechanical system, whose evolution between two subsequent measurements is governed by a unitary operator U. The randomness of the sequence of outcomes, which depends on both the evolution and the measurement under consideration, can be quantified with the help of dynamical entropy (entropy rate) of the corresponding Markov chain generated in the space of measurement outcomes. To quantify the ability of unitary dynamics to produce random sequences of outcomes, we introduce a unitary invariant called quantum dynamical entropy, which is independent of the choice of measurement.

Since the act of measurement can also produce randomness, the value of dynamical entropy is potentially influenced by two independent sources of randomness: the underlying unitary dynamics U and the measurement performed on the system. We eliminate the randomness induced by the measurement by subtracting the dynamical entropy calculated for trivial (identity) dynamics from the original entropy rate calculated for U. Taking the supremum over the suitable class of rank-1 measurements, we obtain the quantum dynamical entropy of U, which is an information-theoretical invariant for the projective unitary group acting on a d-dimensional complex Hilbert space. This allows us to distinguish the class of chaotic unitaries, i.e., the unitary operators that attain the maximum possible value of quantum dynamical entropy $\ln d$.

Chaotic unitaries turn out to be exactly the unitaries that in some orthonormal bases can be represented by appropriately rescaled complex Hadamard matrices. We provide necessary conditions for a unitary operator to be chaotic, expressed in terms of the relation between the trace and the determinant of the operator. As in dimensions two and three these conditions are also sufficient, we compute the volume of the set of chaotic unitaries in the ensemble of unitary matrices in both these dimensions, as well as the average value of quantum dynamical entropy in dimension two. We also prove that the average value of quantum dynamical entropy increases logarithmically with the dimension of the Hilbert space, so the probability that the quantum dynamical entropy of a unitary is almost maximal tends to one as the dimension goes to infinity.

This is joint work with Wojciech Słomczyński.

Open problems on Butson matrices

THU
16:25

Ferenc Szöllősi
Aalto University, Finland

Abstract

In this talk I will mention several open problems related to Butson-type complex Hadamard matrices.

The informational power of the Hoggar SIC-POVM

Anna Szymusiak TUE
Jagiellonian University, Poland 16:25

Abstract

Among positive operator valued measures (POVMs) representing general quantum measurements, symmetric informationally complete (SIC) POVMs play a special role. The eight-dimensional Hoggar lines provide one of the first examples of SIC-POVMs found in dimension larger than two. It seems that this set exhibits a higher level of symmetry than most known SIC-POVMs, and, at the same time, its symmetry has a slightly different character than in case of all other known SIC-POVMs.

The informational power of a quantum measurement is the maximum amount of classical information that can be extracted by the given measurement from any ensemble of quantum states, also equal to the classical capacity of a quantum-classical channel generated by this measurement. In general, it is not easy to compute this quantity analytically, especially in higher dimensions. In the talk I shall show that the informational power of the Hoggar SIC-POVM is equal to $2\ln(4/3)$, the upper bound for informational power of 2-designs, provided recently by Dall'Arno. To this aim the construction of some SIC-POVMs (including the Hoggar lines) from the Hadamard matrices, newly discovered by Jedwab and Wiebe, is used. Moreover, we show that a maximally informative ensemble for a Hoggar SIC-POVM forms another Hoggar SIC-POVM being the image of the original one under a (complex) conjugation, i.e., an antiunitary involutive map.

This is joint work with Wojciech Słomczyński.

Partial permutation decoding for $\mathbb{Z}_{2^k}$-linear Hadamard codes

Mercè Villanueva THU
Universitat Autònoma de Barcelona, Spain 15:10

Abstract

Let $\mathbb{Z}_{2^k}$ be the ring of integers modulo 2^k with $k \geq 1$, and let $\mathbb{Z}_{2^k}^n$ be the set of n-tuples over $\mathbb{Z}_{2^k}$. A nonempty subset $\mathcal{C}$ of $\mathbb{Z}_{2^k}^n$ is a $\mathbb{Z}_{2^k}$-additive code if $\mathcal{C}$ is a subgroup of $\mathbb{Z}_{2^k}^n$. Note that, when $k = 1$, $\mathcal{C}$ is a binary linear code; and when $k = 2$, it is a quaternary linear code or a linear code over $\mathbb{Z}_4$. The $\mathbb{Z}_{2^k}$-additive codes can be seen as binary codes (not necessarily linear) under a generalization of the usual Gray map, $\Phi : \mathbb{Z}_{2^k}^n \rightarrow \mathbb{Z}_2^{n2^{k-1}}$ due to Carlet in 1998. The binary image $C = \Phi(\mathcal{C})$ is a $\mathbb{Z}_{2^k}$-linear code of length $n2^{k-1}$.

Permutation decoding is a technique, first introduced for linear codes, that involves finding a special subset, called a PD-set, of the automorphism group of a code. The question of determining PD-sets has been addressed for several families of linear codes. In 2015, Bernal et al. introduced a new permutation decoding method for $\mathbb{Z}_4$-linear codes and, in general, for systematic codes (not necessarily linear), but the determination of PD-sets for nonlinear codes remained an open problem. Recently, we have established the construction of s-PD-sets of minimum size $s + 1$ for some families of nonlinear systematic codes. We will review some of these results and show the generalization to $\mathbb{Z}_{2^k}$-linear Hadamard codes.

This is joint work with Roland D. Barrolleta.

Congruences for the Number of Transversals of Latin squares

FRI
09:20

Ian Wanless
Monash University, Australia

Abstract

A Latin square of order n is a matrix with n symbols each occurring exactly once per row and once per column. By a diagonal of a Latin square, we mean a set of cells that includes exactly one representative from each row and column. A transversal is a diagonal that contains every symbol. Ryser allegedly conjectured that every Latin square of order n has its number of transversals congruent to n mod 2. The odd half of this conjecture is false, but the even half was proved by Balasubramanian in 1990. We present a number of related results.

Let E_i be the number of diagonals that include exactly i different symbols. So Balasubramanian's theorem is that $E_n \equiv 0 \mod 2$ when $n \equiv 0 \mod 2$. We have a number of new results along the same lines; for example,

- $E_n \equiv 0 \mod 4$ when $n \equiv 2 \mod 4$, and
- $E_{2k} \equiv E_{2k-1} \mod 2$ for $1 \leq k \leq n/2$ when $n \equiv 0 \mod 2$.

We also have a number of conjectures of similar flavour.

Matrix permanents are a key tool in our investigations. Let Λ_n^k be the set of $(0,1)$-matrices of order n with all row and column sums equal to k. We discovered that $\mathrm{per}(A) \equiv 0 \mod 4$ for all $A \in \Lambda_n^k$ when n is odd and $k \equiv 0 \mod 4$ (was this previously known??).

This is joint work with Darcy Best.

Mutually unbiased product bases

TUE
11:15

Stefan Weigert
The University of York, United Kingdom

Abstract

In quantum theory, sets of complex Hadamard matrices arise naturally as so-called mutually unbiased bases which are conceptually important for a number of reasons. They capture, for example, the idea of complementary quantum observables and – if sufficiently many of them exist – allow one to devise elegant and efficient methods to experimentally determine unknown quantum states. However, the required maximal sets of mutually unbiased bases are only known in Hilbert spaces with prime or prime power dimension. The existence of maximal sets in the remaining "composite" dimensions is a long-standing open problem, closely related to the difficulty to list all complex Hadamard matrices in spaces of dimensions such as six or ten etc. Under the restrictive assumption that they contain only product states, we derive tight limits on the number of mutually unbiased bases existing in specific composite dimensions. In some cases, mutually unbiased product bases can be classified exhaustively.

On Near Butson-Hadamard matrices

Oğuz Yayla MON
Hacettepe University, Turkey 15:10

Abstract

Let $\gamma \in \mathbb{R} \cap \mathbb{Z}[\zeta_m]$. In this study we consider γ near Butson-Hadamard matrix that is a generalization of Hadamard matrices. These matrices are examined for non-existence cases in this study. In particular, the unsolvability of certain equations is studied in the case of cyclotomic number fields whose ring of integers is not a principal ideal domain. Winterhof et al. (2014) considered the equations for $\gamma \in \mathbb{Z}$. We first extend this result to $\gamma \in \mathbb{R} \cap \mathbb{Z}[\zeta_m]$ by using some new methods from algebraic number theory. Then, the direct applications of these results to γ-Butson-Hadamard matrices, γ-Conference matrices and nearly perfect sequences are obtained.

Commutators of projectors, mutually unbiased bases and projective geometry

Ilya Zhdanovskiy WED
Moscow Institute of Physics; and Higher School of Economics, Russia 10:05

Abstract

In our article we introduce algebra $\mathcal{N}$ as unital algebra generated by idempotents $p_i, q_j, i, j = 1, ..., 7$ satisfying to relations:

$$p_i p_j = q_i q_j = 0, i \neq j$$

$$p_i q_j p_i = \frac{1}{7} p_i, q_i p_j q_i = \frac{1}{7} q_i$$

and

$$[p_1 + p_2, q_1 + q_2] = [p_3 + p_4, q_3 + q_4] = 0.$$

Last relation was introduced by Nicoara. Using this relation, he proposed another construction of Petrescu family of mutually unbiased bases in dimension 7.

We study algebra $\mathcal{C}$ generated by idempotent P_1, P_2, Q_1, Q_2 with the following relation:

$$[P_1, Q_1] = [P_2, Q_2].$$

It is easy to see that there is a homomorphism: $\mathcal{C} \to \mathcal{N}$ defined by the correspondence:

$$P_1 \mapsto p_1 + p_2, P_2 \mapsto p_3 + p_4; Q_1 \mapsto q_1 + q_2, Q_2 \mapsto q_3 + q_4.$$

I will tell about representation theory of $\mathcal{C}$ and its connection with construction of Petrescu family. Also, I will tell about some generalizations of $\mathcal{C}$ and its relation to projective geometry.

Complex Hadamard matrices with a special structure

TUE
09:20

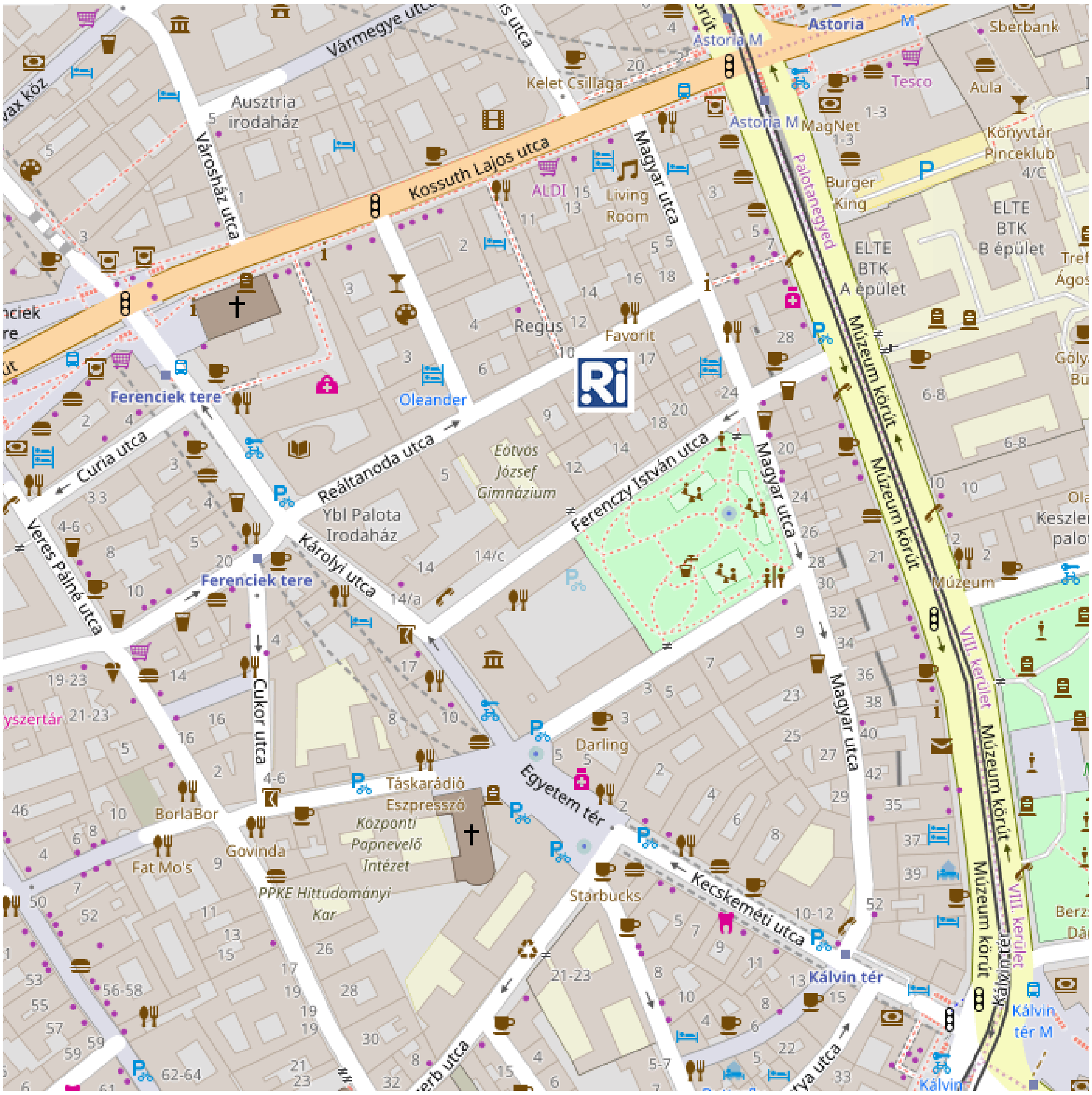
Karol Życzkowski
Jagiellonian University; and Center for Theoretical Physics, Poland

Abstract

We analyze complex Hadamard matrices with special properties, which found some applications in quantum physics. Firstly we discuss *skew complex Hadamard* matrices and address the question for which size they exist. A bistochastic matrix B for which there exist a unitary V, such that $B_{ij} = |V_{ij}|^2$ is called *unistochastic*. Skew complex Hadamard matrices allow us to construct families of *unistochastic* matrices.

Secondly, consider a four index tensor T_{ijkl} of size M. It can be reshaped into a square matrix $A_{\mu\nu}$ of size M^2 with three different choices of composed indices e.g. $\mu = (i, j)$; $\nu = (k, l)$ or $\mu = (i, k)$; $\nu = (j, l)$ or $\mu = (i, l)$; $\nu = (j, k)$. A tensor T is called *perfect* if all three matrices A, A' and A'' generated in this way are unitary. A matrix A is called multiunitary if it remains unitary after suitable reshuffling of their entries. Examples of multiunitary complex Hadamard matrices of size 9 are shown. A question, whether such matrices exist for $N = 36$ is posed, and its relation to the Euler's problem of 36 officers is presented.

Area map (around Rényi Institute)



Good to know: Some of the invited participants are accommodated at Prince Apartments. Street address: Kisfaludy street 18–20, POB 1082, phone: +36 70 310 3740, web: `https://princehotelbudapest.com/`

The street address of Rényi Institute is: Reáltanoda u. 13–15, Budapest.

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