Strong approximation of Black–Scholes theory based on simple random walks

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A basic model in financial mathematics was introduced by Black, Scholes and Merton in 1973 (BSM model). A classical approximation in distribution is the binomial model by Cox, Ross and Rubinstein (1979) (CRR model). The BSM and the CRR models are used for example to price European call and put options. Our aim in this work is to give a strong (almost sure, pathwise) approximation of the BSM model using a suitable nested sequence of simple, symmetric random walks. The approximation extends to the stock price process, the value process, the replicating portfolio, and the greeks. An important tool in the approximation is a discrete version of the Feynman–Kac formula. It is hoped that such a discrete pathwise approximation can be useful for example when teaching students whose mathematical background is limited, e.g. does not contain measure theory and stochastic analysis.

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