Asymptotic results for the generalized allocation scheme

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The generalized allocation scheme was introduced by Kolchin (see [3]). It means that the joint distribution of the random variables $\eta'_1, \ldots, \eta'_N$ is

$$P\{\eta'_1 = k_1, \ldots, \eta'_N = k_N\} = P\Big\{\xi_1 = k_1, \ldots, \xi_N = k_N \ \Big| \ \sum_{i=1}^{N} \xi_i = n\Big\},$$

where $\xi_1, \xi_2, \ldots, \xi_N$ are independent identically distributed non-negative integer valued non-degenerate random variables. This scheme contains several interesting particular cases such as the usual allocation scheme and random forests. Inequalities and limit theorems were proved for Kolchin's generalized allocation scheme in [1] and [2].

Random variables $\eta_1, \ldots, \eta_N$ with joint distribution

$$P\{\eta_1 = k_1, \ldots, \eta_N = k_N\} = P\Big\{\xi_1 = k_1, \ldots, \xi_N = k_N \ \Big| \ \sum_{i=1}^{N} \xi_i \leq n\Big\}$$

were also studied. It can be considered as a general allocation scheme when we place at most n balls into N boxes. In the general allocation scheme the random variable $\mu_{nN} = \sum_{i=1}^{N} I_{\{\eta_i = r\}}$ is the number of boxes containing r balls.

We study laws of large numbers, i.e. the convergence of the average $\frac{1}{N}\mu_{nN}$, as $n, N \to \infty$. We prove local limit theorems, i.e. we study the asymptotic behaviour of $P\{\mu_{nN} = k\}$. We obtain weak limit theorems for the maximum, i.e. we shall consider the asymptotic behaviour of $P\{\max_{1 \leq i \leq N} \eta_i \leq r\}$.

REFERENCES

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[3] Kolchin, V. F., *Random Graphs*, Cambridge University Press, Cambridge, 1991.