

Long range dependence in analysis: Carleson's theorem, dilated series and the Khinchin conjecture

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The theory of weak dependence starts with the investigations of Gauss (1812) on continued fractions and until the 1950's the theory dealt almost exclusively with weak dependence phenomena in analysis and number theory. Starting with the seminal papers of Rosenblatt (1956) and Ibragimov (1962), the theory turned into a purely stochastic direction and today we have a wide and nearly complete theory giving a satisfactory description of the asymptotic properties of weakly dependent systems. Long range dependence, going back to Mandelbrot (1968), Taqqu (1975, 1979), Dobrushin and Major (1979) and others, presents a much harder challenge. Many deep results and problems of analysis, such as Carleson's theorem (1966) on the almost everywhere convergence of trigonometric series, the a.e. convergence of dilated series $\sum_{k=1}^{\infty} c_k f(kx)$, Khinchin's conjecture (1923) of the a.e. convergence of averages $N^{-1} \sum_{k=1}^N f(kx)$ belong to this class. In this lecture, we give a survey of the field and formulate some recent results of the speaker obtained jointly with C. Aistleitner, M. Weber and K. Seip.